Special Session 50: Evolution Equations and Inclusions With Applications to Control, Mathematical Modeling and Mechanics

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The session will focus on the recent developments in the theory of nonlinear evolution equations, optimal control theory and related topics including real life problems of mechanics, biology, economics, and medicine. The main topics of the session include, but are not limited to, analysis of solutions of evolution problems and partial differential equations, operator inclusions, evolution inclusions, control problems, mathematical modeling of natural systems, nonsmooth systems, variational methods, convex and nonconvex problems, optimization of systems and applications.

Rothe method for parabolic variational-hemivariational inequalities

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Xiaoliang Cheng, Piotr Kalita, Yuanjie Yu, Cong Zheng

We deal with the convergence analysis of the semidiscrete Rothe scheme for the parabolic variational-hemivariational inequality with the nonlinear pseudomonotone operator. The problem involves both a discontinuous and nonmonotone multivalued term as well as a monotone term with potentials which assume infinite values and hence are not locally lipschitz. The proof can be viewed both as the proof of solution existence as well as of the convergence of a numerical semidiscrete scheme.

Nonlinear multi-valued reaction-diffusion systems with delay

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Mihai Necula, Daniela Rosu, Ioan I. Vrabie

The aim of this lecture is to present a sufficient condition for the existence of $C^0$-solutions for a class of nonlinear multi-valued reaction-diffusion systems with delay subjected to nonlocal implicit initial conditions. This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

On sensitivity of optimal solutions to control problems for systems governed by evolution subdifferential inclusions

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Krzysztof Bartosz, Piotr Kalita

We consider a class of second order evolution subdifferential inclusions to which can be reduced some hemivariational inequalities. First we quote an existence result for such inclusions based on the theory of pseudomonotone operators. Next we formulate control problems and we present theorems concerning the existence of optimal solutions and the sensitivity under perturbations of state relations as well as the cost functionals. The sensitivity part, based on $\Gamma$-convergence theory, works when we are able to prove the Kuratowski-Painleve convergence of the solution sets for the perturbed state relations and simultaneously the appropriate complementary $\Gamma$-convergence of the perturbed cost functionals to the unperturbed one.

Variational-hemivariational approach to a quasistatic viscoelastic problem with normal compliance, friction and material damage

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Anna Ochal, Meir Shillor

This work studies a model for quasistatic frictional contact between a viscoelastic body and a reactive foundation. The constitutive law is assumed to be nonlinear and contains damage effects modeled by a parabolic inclusion. Contact is described by the normal compliance condition and by a subdifferential frictional condition. A variational-hemivariational formulation of the problem is provided and the existence and uniqueness of its weak solution is proved. The proof is based on a surjectivity result for pseudomonotone coercive operators and a fixed point argument.

Approximate and null controllability results for the heat equation with memory

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The model to be studied is the heat equation with memory in the smooth domain $\Omega \subset \mathbb{R}^m$

$$\frac{\partial w}{\partial t}(t,x) = \Delta_x w(t,x) + \int_0^t M(t-s) \Delta_x w(s,x) ds$$

subject to initial conditions

$$w(0,x) = \xi(x), \quad x \in \Omega$$

and with a boundary control on $\Gamma \subset \partial \Omega$

$$w(t,x) = \begin{cases} v(t,x), & x \in \Gamma, \quad t \in [0,T] \\ 0, & x \in \partial \Omega \setminus \Gamma, \quad t \in [0,T]. \end{cases}$$
or with an interior control
\[
\frac{\partial w}{\partial t}(t, x) = \Delta_x w(t, x) + \int_0^t M(t - s) \Delta_x w(s, x) ds + u(t, x) \chi_{\omega}, \omega \subset \Omega
\]
Under smoothness hypothesis on the kernel \(M\), approximate and null controllability will be investigated using the reduction to a moment problem.

**A global implicit function theorem and its applications**

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The main result of the paper is a global implicit function theorem. In the proof of this theorem, we use a variational approach and apply Mountain Pass Theorem. An assumption guaranteeing existence of an implicit function on the whole space is a Palais-Smale condition. Some applications to integro-differential equations are given.

**Dynamic viscoelastic unilateral contact problem with normal compliance and nonmonotone friction**

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We formulate a dynamic problem that models contact of a viscoelastic body with a rigid foundation covered by a layer of an elastic material. The normal contact is governed, up to a certain threshold, by a normal compliance law and, once this threshold is reached, by a Signorini condition. As for the friction condition, we consider a generalized Tresca law, such that the dependence of the tangential stress on the tangential velocity can be nonmonotone. We provide a proof of the solution existence as well as of the convergence of solutions to the problems with infinite penetration to the one with finite penetration. The results of the numerical experiment are presented as well.

**Estimates for large time controls**

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O. Carja

We study the behavior of the minimum \(L^p\)-norm control, \(p \in (1, \infty]\), needed to drive a linear system to zero as time duration goes to infinity. We apply the result to various examples.

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**Topological methods for semi-linear evolution equations in abstract spaces**

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This talk deals with a semi-linear evolution equation in a Banach space: \(x'(t) = Ax(t) + f(t, x(t))\) which is the abstract formulation of many concrete differential models. The densely defined linear part \(A\) generates a strongly continuous semigroup of contractions; the nonlinear term \(f\) is continuous and possibly superlinear in \(x\). A wide family of nonlocal associated boundary problems is studied, including Periodic, anti-periodic, mean value and multipoint conditions. The investigation is based on topological techniques and suitable Lyapunov-like functions for guaranteeing the required transversality are introduced. Applications to the study of nonlocal population diffusion models complete this discussion.

**Weak and strong solutions to stochastic inclusions and applications**

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In the talk we study the problem of existence of strong and weak solutions to stochastic inclusions. Further we present main topological properties of such solutions and discuss their applications to the theory of fuzzy-valued stochastic equations.

**Existence results for first order evolution inclusions and variational-hemivariational inequalities**

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We deal with an abstract first order evolution inclusion in a reflexive Banach space. The inclusion contains the sum of \(L^p\)-pseudomonotone operator and a maximal monotone operator. We provide an existence theorem which is a generalization of former results known in the literature. Next, we apply our result to the case of nonlinear variational-hemivariational inequalities considered in the setting of an evolution triple of spaces. We specify the multivalued operators in the problem and obtain existence results for several classes of variational-hemivariational inequality problems. Finally, we illustrate our existence result and study a class of quasilinear parabolic problems under nonmonotone and multivalued flux boundary conditions.

**Stochastic delay inclusion with Carathéodory-upper separated multifunctions**

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The existence of strong solutions for stochastic delay
Variational analysis of a diffusion-controlled model for describing the surfactant behavior at the air-water interface

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The study of the dynamic surface tension of surfactant solutions at the air-water interface has been revealed an interesting issue since it plays an important role in several biological, biochemical and industrial processes. When a new surface is formed in a surfactant solution, surfactant molecules migrate from the bulk of the solution to the air-water interface and, consequently, they vary its surface properties. This process is modeled by the partial diffusion equation in one spatial dimension, together with suitable initial and boundary conditions, being the unknowns both the surface and bulk concentrations. Moreover, in order to close the problem, we consider an adsorption model, that is coupled to the system of equations as a boundary condition at the sub-surface. There are two families of models for describing the adsorption dynamics: the diffusion-controlled models and the mixed kinetic-diffusion ones. In this work, we focus on a diffusion-controlled model considering the well-known Langmuir isotherm, for which we study the existence and uniqueness of weak solution. The existence is obtained by using the Rothe method, an intermediate step for proving the existence of Carathéodory-convex selections in nonseparable Banach spaces for multifunctions of the type mentioned above will be discussed in detail.

Bifurcation on a finite time interval in nonlinear hyperbolic-parabolic parameter dependent control system

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The aim of the talk is to discuss doubly nonlinear hyperbolic-parabolic parameter dependent control system. It will be shown that the coupled system of Maxwell’s equation and a heat equation on finite time interval in one space dimension involving a phase transition property and Joule’s heating effect can be written in such a way. This results into two multivalued nonlinearities in the system. Using frequency domain conditions for the hyperbolic-parabolic control system we derive sufficient conditions for bifurcation on a finite time interval. As an important part of the proof we show the existence of a cocycle of our nonautonomous parameter dependent control system.

Existence for a nonlinear delay reaction-diffusion system subjected to nonlocal initial conditions

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We present an existence result for a class of nonlinear delay reaction-diffusion systems subjected to nonlocal initial conditions having affine growth. We also include some sufficient conditions for the uniqueness and global asymptotic stability of the solution and some specific examples. This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011–3-0052.

Consistent high-frequency damping for nonsmooth flexible multibody systems

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This talk is devoted to the consistent numerical integration of nonsmooth flexible multibody systems with impacts and friction. We bring together two ideas at the same time. First, we split non-impulsive and impulsive
force propagation. In the context of time-discontinuous Galerkin methods, we take care of possible impacts and allow velocity jumps at the end of each discretization interval. Second inside discretization intervals, we use sophisticated base integration schemes known from computational mechanics. For non-impulsive periods, the generalized-alpha, ED-alpha (energy-decaying) or Bathe method are motivated by automatically reducing artificial high-frequencies being in the numerical model due to standard space discretization schemes. This technique mixes non-impulsive and impulsive integration strategies, but from the beginning embeds non-impulsive discretizations consistently in a concept which allows velocity jumps and impacts. For the purpose of comparison, the integration schemes are applied to mechanical systems with impacts and Coulomb friction, e.g. imperfect slider-crank type mechanisms. We study convergence, computing time and vibrational behavior by this numerical experiment and discuss the representation of physical oscillations.

Exact controllability of evolution equations by smooth controls and applications

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Let \( V \subset X \subset V' \) are Hilbert spaces with continuous dense injections (see more details in [3] and references therein). Consider the control evolution equation [1]

\[ \dot{x}(t) = Ax(t) + bu(t), \quad x(0) = x^0, \quad 0 \leq t \leq t_0 \]

there exists a square integrable control \( u(\cdot) \in L_2([0,t_1], \mathbb{R}^r) \) such that a mild solution \( x(t, x^0, u(\cdot)) \) of equation with initial condition \( x^0 \), generated by a control \( u(\cdot) \), satisfies the condition \( \|x(t, x^0, u(\cdot))\| \).

On noncompact fractional order differential inclusions with generalized boundary condition and impulses in a Banach space

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We consider a fractional semilinear evolution inclusion in a reflexive Banach space in the presence of impulse effects associated with a very general multivalued boundary condition including several nonlocal conditions as well as the attainability problem. We assume the regularities on the nonlinear terms by means of the weak topology. Thus no compactness is assumed, neither on the evolution operator generated by the linear part, or on the nonlinear term. This technique allows to consider both sublinear and superlinear growth condition on the nonlinear term. The existence of a solution is investigated by means of a fixed point technique. The talk ends with some applications to hyperbolic integro-differential equations arising from physics and biology.

A viability result for delay evolution equations with implicit nonlocal initial conditions

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A necessary and sufficient condition in order that a time-dependent set be viable with respect to a delay evolution equation subjected to an implicit nonlocal initial condition is established. Then, using this result which, as far as we know, is the first one referring to viability for such kind of problems, some concrete applications to nonlinear parabolic equations are derived.

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On the abstract evolution equations of hyperbolic type

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In this talk, we consider existence and uniqueness of classical solution to the abstract Cauchy problem for linear evolution equations of the form

\[ B(t)(d/dt)u(t) + A(t)u(t) = f(t); \quad u(0) = u_0, \]

where \( \{A(t)\} \) and \( \{B(t)\} \) are families of closed linear operators in Hilbert space.

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