Special Session 97: Analysis and Control of Nonlinear Partial Differential Equation Evolution Systems

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It is intended that this Special Session will provide a platform from which renowned specialists in Partial Differential Equations (PDE) and/or Mathematical Control Theory will present their latest research on nonlinear and evolutionary PDE. We anticipate that our speakers will have expertise in a wide-ranging array of topics, possibly including: (i) qualitative and quantitative properties enjoyed by solutions to nonlinear partial differential equations of hyperbolic, parabolic, or of mixed type. Such properties might include global existence and uniqueness; availability of so-called “hidden”, or extra boundary trace regularity; for local well-posedness of solutions, with associated finite time blow-up. In addition, the topic of longtime behavior of solutions for given dissipative PDE could conceivably be broached by one or other of our Speakers, including the existence of attracting sets of finite dimension. (ii) Shape and sensitivity analysis of PDE, particularly with a view toward treating moving boundary phenomena. (iii) Optimization and control problems for nonlinear PDE processes, including the longstanding issue of globally controlling nonlinear hyperbolic PDE. Feedback control schemes to stabilize nonlinear and unstable PDE might also be under discussion in our Special Session, particularly if such schemes are amenable to numerical implementation. The Organizers of this Special Session are hopeful that this bringing together of the various Participants, from various parts of the globe, and each with his or her unique expertise, will spark fruitful discussions and possible future research work in nonlinear PDE control analysis.

Large time behavior of degenerate parabolic equations in domains with non-compact boundary

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We investigate the asymptotic profile as time goes to infinity of solutions to porous-media like equations set in an unbounded open region of $\mathbb{R}^N$, with zero Neumann data prescribed on the boundary. The boundary itself is assumed to be non-compact. We identify the asymptotic profile which in fact is anisotropic due to the shape of the domain itself. We also give an explicit rate of convergence to such a profile, depending again on the shape of the domain. The latter is assumed to satisfy suitable isoperimetric inequalities.

A Mixed Variational Formulation for the Wellposedness and Numerical Analysis of a Certain Fluid-Structure PDE System

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We will present qualitative and numerical results on a partial differential equation (PDE) system which models a certain fluid-structure dynamics. The wellposedness of this PDE model is established by means of constructing for it a nonstandard semigroup generator representation; this representation is essentially accomplished by an appropriate elimination of the pressure. Wellposedness of this fluid-structure dynamics is attained through a certain nonstandard variational (inf-sup) formulation. Subsequently we show how our constructive proof of wellposedness naturally gives rise to a certain mixed finite element method for numerically approximating solutions of this fluid-structure dynamics.

Asymptotic stability for the Schrödinger equation on non compact Riemannian manifolds and exterior domains

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The Schrödinger equation subject to a nonlinear and locally distributed damping, posed in a connected, complete and non compact $n$ dimensional Riemannian manifold $(M,g)$ is considered. Assuming that $(M,g)$ is non-trapping and, in addition, that the damping term is effective in $M\setminus \Omega$, where $\Omega \subset M$ is an open bounded and connected subset with smooth boundary $\partial \Omega$, such that $\overline{\Omega}$ is a compact set, exponential and uniform decay rates of the $L^2$-level energy are established. The main ingredients in the proof of the exponential stability are: (A) an unique continuation property for the linear problem (as in Triggiani and Xu (Contemporary Math./2007)); and (B) a local smoothing effect for the linear and non-homogeneous associated problem (as in Burq, Gerard and Tzvetkov (AIHP/2004))

Control of fourth-order parabolic control systems

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In this talk we address the boundary and internal controllability of some fourth-order parabolic control systems posed on a finite interval. We deal with the null controllability of single and coupled equations by applying the moment theory and a Carleman estimate approach.
Intrinsic decay rate estimates for the wave equation with competing viscoelastic and frictional dissipative effects

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Irena Lasiecka, Marcelo Cavalcanti, Flavio Falcao Nascimento

Wave equation defined on a compact Riemannian manifold \((M,g)\) subject to a combination of locally distributed viscoelastic and frictional dissipations is discussed. The viscoelastic dissipation is active on the support of \(a(x)\) while the frictional damping affects portion of the manifold quantified by the support of \(b(x)\) where both \(a(x)\) and \(b(x)\) are smooth functions. Assuming that \(a(x)+b(x)\geq\delta>0\) for all \(x\in M\) and that the relaxation function satisfies certain nonlinear differential inequality, it is shown that the solutions decay according to the law dictated by the decay rates corresponding to the slowest damping. In the special case when the viscoelastic effect is active on the entire domain and the frictional dissipation is differentiable at the origin, then the overall decay rates are dictated by the viscoelasticity. The obtained decay estimates are intrinsic without any prior quantification of decay rates of both viscoelastic and frictional dissipative effects. This particular topic has been motivated by influential paper of Fabrizio and Polidoro where it was shown that viscoelasticity with poorly behaving relaxation kernel destroys exponential decay rates generated by the linear frictional dissipation. In this paper we extend these considerations to: (i) nonlinear dissipation with unquantified growth at the origin (frictional) and infinity (viscoelastic), (ii) more general geometric settings that accommodate competing nature of frictional and viscoelastic damping. To this end we use an intrinsic method for describing decay rates of the energy via solutions to an appropriate nonlinear ODE system.

Hyperbolic boundary problems with non-traditional boundary conditions

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Sobolev regularity of the solutions to hyperbolic boundary value problems is a persistent issue in many questions concerning the control and stabilization of evolution equations. In the context of shape optimization non-traditional boundary conditions may arise. These boundary conditions satisfy in most cases the Lopatinskii condition but not the Kreiss-Sakamoto (uniform Lopatinski) condition. Hence, the hyperbolic boundary problem is not well-posed in the usual \(L_2\) or Sobolev spaces and a loss of derivatives occurs. We present a survey of the existing literature and show that in most cases the loss of derivatives occurs only in the boundary terms.

Long-time behavior of reaction-diffusion equations with nonlocal boundary conditions on rough domains

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We investigate the long term behavior in terms of finite dimensional global and exponential attractors, as time goes to infinity of solutions to semilinear reaction-diffusion equations on non-smooth domains subject to nonlocal Robin boundary conditions, characterized by the presence of fractional diffusion on the boundary. Our results are of general character and apply to a large class of irregular domains, including domains with a fractal-like geometry. We recover the most of the existing results on existence, regularity, uniqueness, stability, attractor existence, and finite dimension, which are known for the reaction-diffusion equation in smooth domains. The framework we develop also makes possible a number of new results for all diffusion models in other non-smooth settings.

The modified phase field crystal equation

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In recent years, the so-called phase-field crystal (PFC) approach has been recently employed to model and simulate the dynamics of crystalline materials, including, e.g., crystal growth in a supercooled liquid, dendritic and eutectic solidification, epitaxial growth. We consider a modification of the so-called (sixth order) PFC equation introduced by K.R. Elder et al. This variant has recently been proposed by P. Stefanovic et al. to distinguish between elastic relaxation and diffusion time scales. It consists of adding an inertial term (i.e., a second-order time derivative) into the PFC equation. We present some results regarding well-posedness, regularity and long time behavior obtained in collaboration with Hao Wu (Fudan University).

Existence and Stability for Free Liquid Fibers and Films

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In this talk we present recent results on the Matovich-Pearson equations of free liquid fibers and related equations for free liquid films. The governing equations are given by a nonlinear transport equation together with stationary constraints. We will give existence, regularity and stability results for the governing equations, both in the linear and nonlinear regimes, and motivate some open problems.
A new two-component system modelling shallow-water waves

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For propagation of surface shallow-water waves on irrotational flows, we derive a new two-component system of nonlinear partial differential equations. The system is obtained by a variational approach in the Lagrangian formalism. We show that the system has a non-canonical Hamiltonian formulation. We also find its exact solitary-wave solutions.

Strong stability to multiple equilibria in flow structure interactions.

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It is well known that flutter is an endemic phenomenon in aeroelasticity. It occurs in high speed flying jets, suspension bridges, wind mills etc. Eliminating or controlling flutter is one of the fundamental issues arising in applications. From the mathematical point of view, the problem can be modeled by an evolutionary system of coupled PDE’s with an interface. It involves a perturbed wave equation coupled - in a hybrid way- with a nonlinear system of elasticity. It will be shown that the resulting evolutionary system (i) generates a nonlinear semigroup and (ii) the semigroup is strongly stabilizable in the subsonic case. As a consequence, flutter can be eliminated all together in the subsonic regimes. For supersonic velocities it will be shown that the long time behavior of structural solutions is reducible to a finite dimensional attracting set. The above results extend the theory previously known only for the “regularized” models which account for the rotational inertia or thermal effects. The problem may be of interest for applications to traffic flow.

A control problem for a non convex conservation law

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We consider a scalar conservation law in one space dimension
\[ \partial_t u + \partial_x f(u) = 0, \quad t \geq 0, \quad x \in \mathbb{R}, \] (1)
with a smooth flux function \( f \) that suffers a single inflection point. We augment (1) with an initial datum
\[ u(0, x) = \begin{cases} 0 & \text{if } x \notin [a, b] \\ z(x) & \text{if } x \in [a, b] \end{cases} \] (2)
where \( a \) and \( b \) are given real numbers, and \( z = z(x) \) is a bounded measurable function that it is regarded as a control. We are interested in studying the set of attainable profiles at a fixed time \( T > 0 \), i.e. the set
\[ A(T) = \left\{ v \in L^1(\mathbb{R}) : \exists z \in L^\infty(a, b) : v(x) = u(T, x) \text{ a.e.} \right\}. \]

In literature several control problems for conservation laws were studied, but in most cases only strictly convex flux functions were considered. Indeed, the presence of an inflection point changes the structure of the waves in a solution, allowing the presence of one-side contact discontinuities. The problem may be of interest for applications to traffic flow.

Null controllability of degenerate parabolic equations

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We study the null controllability properties of degenerate parabolic equations, when degeneracy occurs at the boundary of the domain. In space dimension 1 and 2, we obtain null controllability results under some necessary and sufficient condition on the degeneracy parameter, providing new global Carleman estimates. In space dimension 1, we are also able to study the behaviour of the controllability cost with respect of the degeneracy parameter, using properties of Bessel functions.

Asymptotic stability for a class of semilinear evolution equations with time delay

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We study the asymptotic behaviour of solutions to an abstract semilinear evolution equation in presence of a time delay in the feedback. It is well-known that a time delay, even arbitrarily small, may destroy the stability of an exponentially stable model. On the other hand, time delay effects are often present in many applications and practical problems. We give an exponential stability result under suitable conditions. Some concrete examples that enter into our abstract framework are also illustrated.

Null controllability for parabolic equations with dynamic boundary conditions of reactive-diffusive type

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We prove null controllability for linear and semilinear heat equations with dynamic boundary conditions of surface diffusion type. The results are based on a new Carleman estimate for this type of boundary conditions.
On the spectral stability of kinks in $\mathcal{PT}$-symmetric Klein-Gordon type models

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We consider the introduction of $\mathcal{PT}$-symmetric terms in the context of classical Klein-Gordon field theories. We explore the implication of such terms on the spectral stability of coherent structures, namely kinks. We find that the conclusion critically depends on the location of the kink center relative to the center of the $\mathcal{PT}$-symmetric term. The main result is that if these two points coincide, the kink’s spectrum remains on the imaginary axis and the wave is spectrally stable. If the kink is centered on the “lossy side” of the medium, then it becomes stabilized. On the other hand, if it becomes centered on the “gain side” of the medium, then it is destabilized. The consequences of these two possibilities on the linearization (point and essential) spectrum are discussed in some detail.

Blow-up for the wave equation with nonlinear source and boundary damping terms

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The talk will deals with blow–up for the solutions of an evolution problem consisting on a semilinear wave equation posed in a bounded $C^{1,1}$ open subset of $\mathbb{R}^n$, supplied with a Neumann boundary condition involving a nonlinear dissipation. The typical problem studied is

$$
\begin{align*}
\partial_{tt} u - \Delta u & = |u|^p u - 2 u & \text{in } [0, \infty) \times \Omega, \\
u & = 0 & \text{on } (0, \infty) \times \Gamma_0, \\
\partial_n u & = -\alpha(x) \left(|u_t|^m u_t + \beta|u|^{\mu-2}u_t\right) & \text{on } (0, \infty) \times \Gamma_1, \\
\nu(0, x) & = u_0(x), & u_t(0, x) = u_t(x) & \text{in } \Omega,
\end{align*}
$$

where $\partial \Omega = \Gamma_0 \cup \Gamma_1$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, $\sigma(\Gamma_0) > 0$, $21$, $\alpha \in L^\infty(\Gamma_1)$, $\alpha \geq 0$, $\beta \geq 0$. The initial data are posed in the energy space. The aim of the talk is to present some recent improvements to previous blow–up results concerning the problem.