Special Session 26: Qualitative Aspects of Nonlinear Boundary Value Problems

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Raul Manasevich, University of Chile, Chile
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In this session we will deal with qualitative aspects of nonlinear boundary value problems such as existence and multiplicity of solutions, dependence upon data and parameters, bifurcation, and topological properties of solutions and global structures. In the case of the periodic boundary value problem, qualitative aspects include Lyapunov stability or instability of solutions. We will consider scalar as well as systems of boundary value problems.

Variational methods for nonlinear perturbations of the mean curvature operator in Minkowski space

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P. Jebelean, J. Mawhin

In this talk we present various existence and multiplicity result for Neumann boundary value problems associated to nonlinear perturbations of the mean curvature operator in Minkowski space. The main tool is Szulkin’s critical point theory for smooth perturbations of convex and lower semi-continuous functionals.

On bound state solutions with a prescribed number of sign changes

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Marta Garcia Huidobro, Cecilia Yarur

We consider radial solutions of the problem

\[ \Delta u + f(u) = 0 \]
\[ \lim_{r \to \infty} u(r) = 0 \]

with a prescribed number of sign changes. Conditions on \( f \) are established that guarantee the existence and the uniqueness of such solutions.

On positive solutions for a class of Caffarelli-Kohn-Nirenberg type equations

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J. Chabrowski

We investigate the solvability of singular equations of Caffarelli-Kohn-Nirenberg type having a critical-like nonlinearity with a sign-changing weight function.

Boundary blow up of nonnegative solutions of some elliptic systems

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We study the nonnegative solutions of the elliptic system

\[ \Delta u = |x|^a v^\delta, \quad \Delta v = |x|^b u^\mu, \]

where \( a, b \) are real numbers such that \( a, b > \max\{-2, -N\} \), in the superlinear case \( \mu \delta > 1 \), which blow up near the boundary of a domain of \( \mathbb{R}^N \), or at one isolated point. In the radial case we give the precise behavior of the large solutions near the boundary in any dimension \( N \). We also show the existence of infinitely many solutions blowing up at 0. Furthermore,
we show that there exists a global positive solution in $\mathbb{R}^N \setminus \{0\}$, large at 0, and we describe its behavior. We apply the results to the sign changing solutions of the biharmonic equation

$$\Delta^2 u = |x|^\beta |u|^\alpha.$$  

Our results are based on a new dynamical approach of the radial system by means of a quadratic system of order 4. This talk is part of a joint work with Prof. M. Véron and Prof. C. Yarur.

A mean curvature type of geometric parabolic equation

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Pengfei Guan  
In this talk, we will present a parabolic PDE defined on hypersurfaces and its fully non-linear analogue. For any closed star-shaped smooth hypersurface, this flow exists for all time $t>0$ and exponentially converges to a round sphere. Moreover, we will show that all the quermass integrals evolve monotonically along this flow. Consequently, we prove a class of isoperimetric type of inequalities including the classical isoperimetric inequality on star-shaped domains. We will also present a fully non-linear analogue of this flow. More specifically, we study a fully non-linear parabolic equation of a function on the standard sphere and discuss its long-time existence and exponential convergence. As applications, we recover the well-known Alexandrov-Fenchel inequalities on bounded convex domains in Euclidean space.

$\rightarrow \infty \circ \infty \leftarrow$

Convergence versus periodicity in a dynamical system arising in the study of a higher-order elliptic PDE

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Paul G. Schmidt  
Trying to understand the blow-up behavior of large radial solutions of a polyharmonic PDE with power nonlinearity, we are led to analyze the dynamics of a parameter-dependent single-loop positive-feedback system; the dimension of the system corresponds to the order of the PDE. In low dimensions, we observe convergence to equilibrium; in high dimensions, multiple periodic orbits arise via successive Hopf bifurcations. We discuss the dynamics of the system and consequences for the underlying PDE.

$\rightarrow \infty \circ \infty \leftarrow$

On the energy of the current vector of a complex valued function in $\mathbb{R}^3$

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For a complex valued function $u : \Omega \subset \mathbb{R}^3 \to \mathbb{C}$, the current vector is usually defined as $j(u) = \text{Im}(\bar{u} \nabla u)$. In this talk I will present an estimate for $\int_{\Omega} |j(u)|^2$ in terms of quantities determined by a given curve $\Gamma$ in $\Omega$. These estimates are valid when $\nabla \times j(u)$ and $\Gamma$ are sufficiently close as functionals on vector fields.

$\rightarrow \infty \circ \infty \leftarrow$
Eigenvalue-curves and nonlinear second order elliptic equations with nonlinear boundary conditions

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N. Mavinga
We are concerned with the existence of eigenvalue-curves connecting the Steklov and Neumann-Robin spectra of linear second order elliptic equations. We then discuss nonlinear elliptic problems with nonlinear boundary conditions when the reaction and boundary nonlinearities stay in some sense between two consecutive eigenvalue-curves.

\[ \longrightarrow \infty \circ \infty \longleftrightarrow \]

Nonlinear periodic boundary value problems via initial value problems: generalized quasilinear techniques

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J. O. Adeyeye, D. H. Dezern
It is well known that the Monotone Iterative (MIT) and the Generalized Quasilinear Technique (GQT) are two important techniques for obtaining solutions, in a constructive way, of a variety of nonlinear Initial value Problems (IVPs) and Periodic Boundary Value Problems (PBVPs). Whereas the rate of convergence of the iterates in the former technique is linear, that in the latter is quadratic, and hence more rapid. In this talk, we present a new approach to solve nonlinear PBVPs by employing the GQT. The fact that the iterates in our approach are solutions of (linear) IVPs (as opposed to PBVPs, in the conventional approach), and converge to the multiple solutions (or the unique solution, under additional conditions) of the given nonlinear PBVP, allows us to dispense with many a hypothesis required in the conventional approach. We provide graphical and numerical examples to support the results in our new approach.

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Hyperbolic fractional Laplacian

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V. Banica, M.d.M. González, M. Sáez
In this talk I will discuss an appropriate definition for the fractional Laplacian on Hyperbolic Space. In the spirit of the work of L. Caffarelli and L.Silvestre, we show that we can associate an extension problem to our definition.

\[ \longrightarrow \infty \circ \infty \longleftrightarrow \]

Oscillatory entire solutions of polyharmonic equations with subcritical growth

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The following two-point boundary value problem is considered:

\[ u'' + k(x)f(u) = 0 \text{ on } (-1, 1); \quad u(-1) = u(1) = 0. \]

Here, \( h \in C[-1, 1] \cup C^1([-1, 1] \setminus \{0\}), \)
\( h(x) > 0 \text{ on } [-1, 1] \setminus \{0\}, \quad f \in C^1[0, \infty), \quad f(s) > 0 \text{ for } s > 0 \) and \( f(0) = 0. \) In addition, the superlinear condition \( sf'(s) > f(s) \) on \((0, \infty)\) is satisfied. In many cases, the positive solution of this problem is unique. Indeed, if \( h(x) = 1, \) then the problem has at most one positive solution. However, for example, if \( f(s) = s^p, \quad p > 1, \quad h(x) = |x|^l \) and \( l > 0 \) is sufficiently large, then there exist three positive solutions. In this talk, a sufficient condition is derived for the existence of three positive solutions.

\[ \longrightarrow \infty \circ \infty \longleftrightarrow \]

On some connecting orbits for a class of singular second order Hamiltonian systems

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We present some existence results for connecting orbits emanating from 0 of a class of singular second order Hamiltonian equations of the form

\[ u'' + V(t, u) = 0, \quad t \in \mathbb{R} \]

where the singular potential \( V(t, u) \) is \( T \) periodic and satisfies Strong-Force condition at the singularity.

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