

STABILIZED FINITE ELEMENT METHOD FOR THE NON-STATIONARY NAVIER-STOKES PROBLEM

YINNAN HE

Faculty of Science, Xi'an Jiaotong University
Xi'an 710049, P. R. China

YANPING LIN

Department of Mathematics Science, University of Alberta,
Edmonton, Alberta., Canada T6G 2G1

WEIWEI SUN

Department of Mathematics, City University of Hong Kong,
83 Tat Chee Avenue, Kowloon, Hong Kong

(Communicated by Shouhong Wang)

ABSTRACT. In this article, a locally stabilized finite element formulation of the two-dimensional Navier-Stokes problem is used. A macroelement condition which provides the stability of the $Q_1 - P_0$ quadrilateral element and the $P_1 - P_0$ triangular element is introduced. Moreover, the H^1 and L^2 -error estimates of optimal order for finite element solution (u_h, p_h) are analyzed. Finally, a uniform H^1 and L^2 -error estimates of optimal order for finite element solution (u_h, p_h) is obtained if the uniqueness condition is satisfied.

1. Introduction. The development of appropriate mixed finite element methods is a key component in the search for efficient techniques for solving the incompressible Navier-Stokes problem. Using a primitive variable formulation, the importance of ensuring the compatibility of the component approximations of velocity and pressure by satisfying the so-called inf-sup condition is widely understood. It is also well known that the simplest conforming low-order elements like the $P_1 - P_0$ (linear velocity, constant pressure) triangular elements are not stable. This impinges on efficiency, since the simple logic and regular data structure associated with low-order finite element methods makes them particularly attractive on modern vector and parallel processing architectures.

The stability of the mixed approximations has become crucially important with the advent of “fast” iterative solution algorithms, for example, based on multigrid or preconditioned conjugate gradient iterations. Numerical experiments show that in the solution of the Stokes or Navier-Stokes problems, ensuring stability is essential if a reasonable rate of convergence of such iterations is to be achieved. For details,

2000 *Mathematics Subject Classification.* Primary: 35L70, 65N30, 76D06.

Key words and phrases. Navier-Stokes problem, stabilized finite element, uniform error estimate.

This work is supported in parts by the NSF of China 10371095, NSERC of Canada and the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CityU 102103).