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QUALITATIVE BEHAVIOR OF A FAMILY OF DELAY-DIFFERENTIAL MODELS OF THE GLUCOSE-INSULIN SYSTEM

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Abstract. A family of delay-differential models of the glucose-insulin system is introduced, whose members represent adequately the Intra-Venous Glucose Tolerance Test and allied experimental procedures of diabetological interest. All the models in the family admit positive bounded unique solutions for any positive initial condition and are persistent. The models agree with the physics underlying the experiments, and they all present a unique positive equilibrium point.

Local stability is investigated in a pair of interesting member models: one, a discrete-delays differential system; the other, a distributed-delay system reducing to an ordinary differential system evolving on a suitably defined extended state space. In both cases conditions are given on the physical parameters in order to ensure the local asymptotic stability of the equilibrium point. These conditions are always satisfied, given the actual parameter estimates obtained experimentally. A study of the global stability properties is performed, but while from simulations it could be conjectured that the models considered are globally asymptotically stable, sufficient stability criteria, formally derived, are not actually satisfied for physiological parameters values. Given the practical importance of the models studied, further analytical work may be of interest to conclusively characterize their behavior.

1. **Introduction.** The modeling of the glucose-insulin system is an appealing and challenging topic in biomathematics and many different models have been presented in the last decades, mostly referred to the experimental framework of the *Intra Venous Glucose Tolerance Test* (IVGTT), where a bolus of glucose is administered intra-venously and glucose and insulin concentrations are frequently sampled (see e.g. the ODEs of the Minimal Model [1], [17], or the more recent integro-differential equations models of [2], [10], [12]). An interesting survey on a very wide class of most significant models available in the literature and the software tools related to them can be found in [11]. The interest in modelling this physiological system stems in part from its relative simplicity, in part from the social impact of its derangement (giving rise to the widespread disease Diabetes Mellitus), in part from the actual

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