

## MULTIPLE POSITIVE PERIODIC SOLUTIONS FOR A DELAY HOST MACROPARASITE MODEL

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(Communicated by Carmen Chicone)

**ABSTRACT.** A scalar non-autonomous periodic differential equation with delays arising from a delay host macroparasite model is studied. Two results are presented for the equation to have at least two positive periodic solutions: the hypotheses of the first result involve delays, while the second result holds for arbitrary delays.

**1. Introduction.** In recent years, periodic population dynamics with delays has become a very popular subject, and various models have been studied (See [1, 6, 8, 9, 10, 12, 13] and the references therein). One of the important mathematical problems for such models is to show the existence of positive periodic solutions. For example, the scalar equation with a delay  $r$

$$x'(t) = a(t)x(t) \left[ \frac{1}{(1 + \kappa x(t-r))^n} - \frac{1}{(1 + \kappa_1 x(t-r))^n} - c(t) \right] \quad (1.1)$$

arises from a delay host macroparasite model (See [10, 12] and the references therein), where  $\kappa < \kappa_1$  and  $r$  are positive numbers,  $n$  is a positive integer,  $a$  and  $c$  are continuous and positive  $\omega$ -periodic functions on  $(-\infty, \infty)$ ,  $c$  is not identically constant, and  $x$  is the number of sexually mature worms in the human community of some fixed size. Let

$$f(x) = \frac{1}{(1 + \kappa x)^n} - \frac{1}{(1 + \kappa_1 x)^n}, \quad \text{and} \quad x^* = \frac{\kappa_1^{\frac{1}{n+1}} - \kappa^{\frac{1}{n+1}}}{\kappa_1 \kappa^{\frac{1}{n+1}} - \kappa \kappa_1^{\frac{1}{n+1}}}.$$

We see that  $f > 0$  on  $(0, \infty)$ ,  $f(0) = \lim_{x \rightarrow \infty} f(x) = 0$  and  $f$  has a unique maximum at  $x^*$ . Lemma 2.3 shows that if  $\max_{0 \leq t \leq \omega} c(t) < f(x^*)$  and  $r = 0$ , then (1.1) has two (and only two) positive  $\omega$ -periodic solutions, one lying in  $(0, x^*)$  and the other lying in  $(x^*, \infty)$ . The goal of the paper is to show that, under some further conditions, (1.1) has (at least) two positive  $\omega$ -periodic solutions for  $r \neq 0$ . Roughly speaking, we present two such results: Theorems 2.1 and 3.1. Theorem 2.1 holds for “small”  $r$ , while Theorem 3.1 holds for all  $r$ .

In fact, we study a more general equation:

$$x'(t) = a(t)g(t, x(t - r_0(t)))[f(x(t - r(t))) - c(t)], \quad (1.2)$$

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2000 *Mathematics Subject Classification.* 34K13, 92B05, 92D25.

*Key words and phrases.* Delay host macroparasite model, multiple positive periodic solutions, Schauder’s and Krasnosel’skii’s fixed point theorems.