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WAVELET CONSTRUCTIONS IN NON-LINEAR DYNAMICS

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Abstract. We construct certain Hilbert spaces associated with a class of non-linear dynamical systems X. These are systems which arise from a generalized self-similarity and an iterated substitution. We show that when a weight function W on X is given, then we may construct associated Hilbert spaces $H(W)$ of L^2-martingales which have wavelet bases.

1. Introduction

A particularly productive approach to the construction of wavelet bases in $L^2(\mathbb{R})$ is based on the notion from optics of *resolution*, which translates into scales of nested Hilbert subspaces V_n, $n \in \mathbb{Z}$, in $L^2(\mathbb{R})$ such that the intersection is $\{0\}$ and the union is dense. Moreover the operation of dyadic scaling transforms each V_n to the next V_{n+1}. This is called a multiresolution approach to wavelets (see [Dau92]), and it is based on the interplay between the two abelian groups $\mathbb{T}$ (the circle group = one-torus), and $\mathbb{R}$ (the real line), with $\mathbb{T}$ representing a period interval placed on the line $\mathbb{R}$. This paper is based on the observation that multiresolutions really are martingales; and by exploiting this fact, we are able to adapt the geometric idea of subspaces (V_n) to nonlinear dynamics in a variety of applications where such a pair of groups is not available, but instead there is a single endomorphism on a compact space X which defines a certain self-similarity mirroring the more familiar scale-similarity that is so powerful in wavelet analysis.

The words 'non-linear' and 'wavelets' in the title beg two questions: (1) "What is the Hilbert space?" (2) If 'non-linear', then there must be a substitute for the duality between the operators of translation and multiplication!?

We will address the questions in the announcement below, while giving answers with full proofs in the forthcoming papers [DJ03, DJ04a, DJ04b, DJ04c].

We have in mind three classes of examples: (1) The state space X for a subshift system in symbolic dynamics; (2) affine iterated function systems based on a fixed expansive scaling matrix; and (3) the complex iteration systems which generate Julia sets X in the Riemann sphere. If $r(z)$ is a rational function, set

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