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RECENT PROGRESS ON THE BOUNDARY RIGIDITY PROBLEM

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Abstract. The boundary rigidity problem consists in determining a compact, Riemannian manifold with boundary, up to isometry, by knowing the boundary distance function between boundary points. In this paper we announce the result of our forthcoming article that one can solve this problem for generic simple metrics. Moreover we probe stability estimates for this problem.

1. Main results

Let $(M, \partial M, g)$ be a compact Riemannian manifold with boundary. Denote by ρ_g the distance function in the metric g. We consider the inverse problem of whether $\rho_g(x, y)$, known for all x, y on ∂M, determines the metric uniquely. It is clear that any isometry which is the identity at the boundary will give rise to the same distance functions on the boundary. Therefore, the natural question is whether this is the only obstruction to uniqueness. This is known in differential geometry as the boundary rigidity problem. The boundary distance function only takes into account the shortest paths, and it is easy to find counterexamples where ρ_g does not carry any information about certain open subset of M, so one needs to pose some restrictions on the metric. One such condition is simplicity of the metric.

Definition 1.1. We say that the Riemannian metric g is *simple* in M, if ∂M is strictly convex with respect to g, and for any $x \in M$, the exponential map $\exp_x : \exp_x^{-1}(M) \to M$ is a diffeomorphism.

Michel [13] conjectured that a *simple* metric g is uniquely determined, up to an action of a diffeomorphism fixing the boundary, by the boundary distance function $\rho_g(x, y)$ known for all x and y on ∂M.

This problem also arose in geophysics in an attempt to determine the inner structure of the Earth by measuring the travel times of seismic waves. It goes back to Herglotz [11] and Wiechert and Zoeppritz [30]. Although the emphasis has been on the case that the medium is isotropic, the anisotropic case has been of interest in geophysics since it has been found that the inner core of the Earth exhibits anisotropic behavior [6].

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