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INVARIANT MEASURES FOR THE HOROCYCLE FLOW ON PERIODIC HYPERBOLIC SURFACES

FRANÇOIS LEDRAPPIER AND OMRI SARIG

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Pour Martine

Abstract. We describe the ergodic invariant Radon measures for the horocycle flow on general (infinite) regular covers of finite volume hyperbolic surfaces. The method is to establish a bijection between these measures and the positive minimal eigenfunctions of the Laplacian of the covering surface.

1. Introduction

Let $\mathbb{D}$ be the open unit disc equipped with the hyperbolic metric $2|dz|/(1-|z|^2)$, and let $T^1(\mathbb{D})$ denote the collection of unit tangent vectors to $\mathbb{D}$. A *horocycle* in $\mathbb{D}$ is a circle contained in $\overline{\mathbb{D}}$ which touches $\partial\mathbb{D}$ at one point. For every $v \in T^1(\mathbb{D})$ there is a unique horocycle which passes through the base point of v and the terminus of geodesic determined by v. The *horocycle flow* of $\mathbb{D}$ is the flow $h^t : T^1(\mathbb{D}) \to T^1(\mathbb{D})$ which moves a unit tangent vector along the horocycle it determines in the positive direction, at unit speed.

A hyperbolic surface can be written in the form $M := \Gamma\backslash\mathbb{D}$ where Γ is a torsion free discrete subgroup of $\text{Möb}(\mathbb{D})$, the group of Möbius transformations of the disc. The horocycle flow and geodesic flow descend to flows h, g on $T^1(M)$.

A famous theorem of Furstenberg [F] says that if M is compact, then h has up to normalization a unique invariant Radon measure. Variants of this phenomena have been established for more general geometrically finite hyperbolic surfaces by Dani [D], Burger [Bu] and Roblin [Ro], and for more general actions by Ratner [Ra]. The geometrically infinite case is still almost completely open.

We contribute to the understanding of this case by treating the *periodic surfaces*

$$M = \Gamma\backslash\mathbb{D} \text{ where } \{id\} \neq \Gamma \lhd \Gamma_0,\ \Gamma_0 \text{ is a torsion free lattice in } \text{Möb}(\mathbb{D}).$$

M is a regular cover of the finite volume surface $M_0 = \Gamma_0\backslash\mathbb{D}$. We call M_0 the *period* of M. The group of deck transformations G is called the *symmetry group* of M. A periodic surface is called *cocompact* if M_0 is compact.

The symmetry group is always finitely generated, because $G \simeq \Gamma_0/\Gamma$ and Γ_0 is finitely generated. Any finitely generated group can be realized as a symmetry

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