THE $\mathrm{SL}(2,\mathbb{C})$ CHARACTER VARIETY OF A ONE-HOLED TORUS

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ABSTRACT. In this note we announce several results concerning the $\mathrm{SL}(2,\mathbb{C})$ character variety $\mathcal{X}$ of a one-holed torus. We give a description of the largest open subset $\mathcal{X}_{BQ}$ of $\mathcal{X}$ on which the mapping class group Γ acts properly discontinuously, in terms of two very simple conditions, and show that a series identity generalizing McShane's identity for the punctured torus holds for all characters in this subset. We also give variations of the McShane-Bowditch identities for characters fixed by an Anosov element of Γ with applications to closed hyperbolic three-manifolds. Finally we give a definition of end invariants for $\mathrm{SL}(2,\mathbb{C})$ characters and give a partial classification of the set of end invariants of a character in $\mathcal{X}$.

1. INTRODUCTION

Let T be a one-holed torus, π its fundamental group, and $\Gamma := \pi_0(\mathrm{Homeo}(T))$ the mapping class group of T. In this note we announce several results concerning the $\mathrm{SL}(2,\mathbb{C})$ character variety $\mathcal{X}$ of T. Here is a brief description of our results. We first give a characterization of the largest open subset $\mathcal{X}_{BQ}$ of $\mathcal{X}$ on which the mapping class group Γ acts properly discontinuously, in terms of two very simple conditions, called the Bowditch Q-conditions. This generalizes results of Bowditch [5], who gave a similar description for the "type-preserving" characters, and also of Goldman [7] (see also [6]), who studied the dynamics of the action of Γ on the *real* $\mathrm{SL}(2)$ characters, and gave a (geometric) description of the set for the real characters. Note that for $[\rho] \in \mathcal{X}$, it is possible to verify algorithmically whether $[\rho]$ satisfies these conditions.

We next show that a series identity generalizing McShane's remarkable identity for the punctured torus holds for all characters in this subset (the original identity can be regarded as the formal derivative of the general identity evaluated at an appropriate parameter value). This generalizes results of McShane [11], [12], Bowditch [3], [5], Mirzakhani [13], and the authors [16], [17] for two-generator subgroups of $\mathrm{SL}(2,\mathbb{C})$. We also give necessary and sufficient conditions for this identity to hold for characters $[\rho] \in \mathcal{X}$, thereby giving a complete answer to the question of when the identity holds for two-generator subgroups of $\mathrm{SL}(2,\mathbb{C})$.