Special Session 37: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

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This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include reaction-diffusion equations, fluid dynamics, delay, localized, nonlocal, degenerate evolution equations, steady states and their properties.

**Hyperbolic equations with variable nonlinearity: existence and blow-up**

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We study the Dirichlet problem for a class of nonlinear hyperbolic equations with $p(x,t)$-Laplacian relatively spatial variables and with damping term. Under suitable conditions on the data, we prove local and global existence theorems and study the finite time blow-up of the energy solutions. Also we consider Young measure solutions of such equations. The analysis relies on the methods developed in [1-6].

**References**


**Convergence to Steady State for Degenerate Parabolic Equations**

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In this talk, we first discuss large time behavior of the homogeneous Dirichlet problem to the degenerate parabolic equation $u_t = g(u)\Delta u + f(u)$ in a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$. Under suitable conditions on $f(u)$ and $g(u)$, we show that all solutions will converge to the steady state exponentially. Next, we study the degenerate parabolic system $u_t = \Delta u + u(\partial_1 - b_1 u + c_1 v)$ and $v_t = v(\Delta u + v(\partial_2 + b_2 u - c_2 v))$ with the same boundary condition. We show that any positive solutions converge to a unique steady state exponentially if the coefficients satisfy some conditions.

**Blow-up for possible singular solutions of the Navier-Stokes equations**

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In this talk we will present blow-up rates for possible singular solutions to the Navier-Stokes equations in the homogeneous Sobolev spaces $\tilde{H}^2(X)$ and $\tilde{H}^2(\mathbb{R}^3)$, where $X = \mathbb{T}^3$, the 3-dimensional torus, or $X = \mathbb{R}^3$.

**Global in time weak solutions for a nonlinear model for tumor growth.**

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We investigate a free boundary problem modeling the growth of tumors cells. The model is given by a multi-phase flow and the tumor is described as a growing continuum $\Omega$ with boundary $\partial \Omega$ both of which evolve in time. In particular the model consists of a nonlinear second-order parabolic equations describing the diffusion of nutrient, and three nonlinear first-order hyperbolic equations describing the evolution of proliferative, quiescent cells and dead cells. Global-in-time weak solutions are obtained using an approach based on penalization of the boundary behavior, diffusion and viscosity in the weak formulation.

**Problems with singularity in the u variable: nonnegative solutions**

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We deal with the existence of nonnegative solutions to parabolic problems which are singular in the $u$ variable whose model is

\[
\begin{aligned}
&u_t - \Delta p u = f(x,t)(\frac{1}{\theta} + 1) &\text{in } \Omega \times (0,T) \\
&u(x,t) = 0 &\text{on } \partial \Omega \times (0,T) \\
&u(x,0) = u_0(x) &\text{in } \Omega.
\end{aligned}
\]

Here $\Omega$ is a bounded open subset of $\mathbb{R}^N$, $N \geq 2$, $0 < T < +\infty$, $\theta > 0$, $\Delta p u = -\text{div}(\nabla u^{p-1} \nabla u)$ with $p > 1$. As far as the data, we assume $f(x,t) \in L^p(0,T;L^p(\Omega))$. 

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with $\frac{1}{q} + \frac{1}{p} < 1$, $f(x,t) \geq 0$ a.e. in $\Omega \times (0,T)$ and $u_0(x) \geq 0$ a.e. in $\Omega$. We consider also the case where the right hand side depends on the gradient of the solution. In this last case the model of the right hand side is $F(x,t,u,\nabla u) = \frac{(\nabla u)^q}{|\nabla u|^q}$, with $\theta > 0$, $D > 0$, $1 < q < p$ and $f(x,t)$ as before.

**Analysis on the initial-boundary value problem of a full bipolar hydrodynamic model for semiconductors**

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In this paper, we study the initial boundary value problem of the one dimensional full bipolar hydrodynamic model for semiconductors. The existence and uniqueness of the stationary solution are established by the contraction mapping theorem. The exponentially asymptotic stability of the stationary solution is given by means of the energy estimate method.

**Existence and blow-up of solutions for semilinear filtration problems**

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We first examine the local existence and uniqueness of solutions $u = u(x,t,\lambda)$ to the semi linear filtration equation $u_t = \Delta K(u) + \lambda f(u)$, for $\lambda > 0$, with initial data $u_0 \geq 0$ and appropriate boundary conditions. Our main result is the proof of blow-up of solutions for some $\lambda$. Moreover, we discuss the existence of solutions for the corresponding steady-state problem. It is found that there exists a critical value $\lambda^*$ such that for $\lambda > \lambda^*$ the problem has no stationary solution of any kind, while for $\lambda \leq \lambda^*$ there exist classical stationary solutions. Finally, our main result is that the solution $u_\lambda$ for $\lambda > \lambda^*$ blows-up in finite time independently of $u_0 \geq 0$. The functions $f,K$ are mostly positive, increasing and convex and $K'/f$ is integrable at infinity.

**Existence and stability of traveling waves for an Allen-Cahn model with relaxation**

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We investigate an hyperbolic variation of the Allen–Cahn equation (bistable reaction–diffusion equation), where the Fick’s law of diffusion is replaced by a relaxation term, thus introducing a delay in the process. The main feature of the model is the combination of the dissipation coming from the relaxation term, which play the role of the diffusive transport mechanism of the classical Allen–Cahn equation, and a zero-order reactive term, which determines the presence of two stable constant states.

Some rigorous results concerning existence and stability of traveling waves for this hyperbolic model are provided, together with numerical experiments, also in connection with the standard parabolic Allen–Cahn equation.

**Smooth Solutions to Strongly Coupled Elliptic Systems on 2D Domains**

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We discuss the existence of smooth solutions to a class of strongly coupled elliptic systems consisting of two or more equations. Since maximum principles for such systems are not available, the solutions are not known to be bounded a priori and the ellipticity constants can be unbounded. The theories of BMO functions and $A_p$ weights are used here to bypass these obstacles to provide the existence of classical solutions to the systems.

**On a hyperbolic equation in MEMS**

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We consider a damped wave equation with singular nonlinearity and Dirichlet boundary condition in a bounded domain, which describes an electrostatic micro-electromechanical system (MEMS) device. We show that the pull-in voltage $\lambda^*$ is the critical threshold for global existence and quenching in this wave equation. More precisely, if the applied voltage $\lambda^*$, then any solution quenches in finite time. Finally, we analyze the relation between the hyperbolic model and the parabolic model through the viscosity dominated limit.

**Numerical Results on Asymptotic Stability of Travelling Wave for Nicholson’s Blowflies Equation**

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Ming Mei, Chi-Kun Lin, Yanping Lin

In this talk, we numerically study the asymptotic stability of travelling wave solutions for Nicholson’s Blowflies equation, a time-delayed reaction diffusion equation, with local or nonlocal nonlinearity. It is known that, when the ratio of birth rate coefficient and death rate coefficient $p/d$ lies between 1 and $e$, the equation is monotone and possesses monotone traveling wave solutions. However, when the rate is larger than $e$, the equation loses its monotonicity and may possess non-monotone traveling waves when the delay time $r$ is large, which causes the study of stability of these non-monotone traveling waves to be challenging. In this talk, for the case $p/d$ lies between $e$ and $e^2$, we numerically show that monotone and non-monotone travelling waves are exponentially stable. For the case that $p/d$ is larger than $e^2$, monotone or non-monotone travelling waves are exponentially stable for some small delay time, and are unstable for large delay time. Several interesting numerical results will be
demonstrated too. Joint work with Ming Mei, Chi-Kun Lin and Yanping Lin.

**Long-time dynamics of a thermoelastic plate with second sound**

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We consider a nonlinear thermoelastic plate where the heat flux is given by the Cattaneo’s law. In the presence of a rotational inertia term, we prove the existence of a finite-dimensional global attractor under the sole dissipation given by the heat flux. On the other hand, without rotational inertia, the existence of a global attractor is done by adding a frictional damping in the displacement equation.

**Metastable dynamics in conservation laws**

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The presence of small parameters, such as viscosity coefficients or relaxation time, induces in the dynamics of evolutive PDEs the presence of different timescales. As a consequence, systems may exhibit an initial fast transient leading the solutions close to a low-dimensional approximately invariant manifold, followed by a slow evolution in the vicinity of the manifold itself. The aim of this talk is to present the mathematical quantification of the slow motion relative to the long-time scale, starting from the basic example of viscous scalar conservation laws and discussing some possible extensions to more realistic models.

**Stationary solutions to symmetric hyperbolic-parabolic systems in half space**

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In this talk, we consider the large-time behavior of solutions to hyperbolic-parabolic coupled systems in the half line. Assuming that the systems admit the entropy function, we may rewrite them to symmetric forms. For these symmetrizable hyperbolic-parabolic systems, we first prove the existence of the stationary solution. In the case where one eigenvalue of Jacobian matrix appeared in a stationary problem is zero, we assume that the characteristics field corresponding to the zero eigenvalue is genuine non-linear in order to show the existence of a degenerate stationary solution. We also prove that the stationary solution is time asymptotically stable under a smallness assumption on the initial perturbation. The key to the proof is to derive the uniform a priori estimates by using the energy method in half space developed by Matsumura and Nishida as well as the stability condition of Shizuta-Kawashima type. These theorems for the general hyperbolic-parabolic system cover the compressible Navier-Stokes equation for heat conductive gas.

**Solvability and long time behavior of nonlinear Reaction-Diffusion equations with Robin Boundary Condition**

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We investigated the long-time behavior and solvability of the reaction-diffusion equation, which has a polynomial growth nonlinearity of arbitrary order, with Robin boundary condition on the bounded domain. The problem that we investigate as the following:

\[
\begin{aligned}
&u_t - \Delta u + a(x,t)|u|^{p-1}u - b(x,t)|u|^q u = h(x,t), \\
&(x,t) \in Q_T \\
&\left.\left(\frac{\partial u}{\partial \eta} + k(x',t)u\right)\right|_{\partial \Omega} = \varphi(x',t), \\
&(x',t) \in \Sigma_T \\
&u(x,0) = u_0(x), x \in \Omega,
\end{aligned}
\]

here \( \Omega \subset \mathbb{R}^n \) \((n \geq 3)\), is a bounded domain such that \( \partial \Omega \) sufficiently smooth boundary, \( T > 0, Q_T = \Omega \times (0,T) \) and \( \Sigma_T = \partial \Omega \times [0,T] \). In super linear case, for the existence and uniqueness of the generalized solution of problem (1)-(3), we obtain sufficient conditions for functions \( a, b \) and \( k \) and relations between \( p \) and \( q \) and under these conditions we show the existence of generalized solution of problem (1)-(3) and the uniqueness of the solution in corresponding spaces, by applying a general existence theorem. For the long-time behavior, firstly we prove that solution has an absorbing set in \( L^2(\Omega) \). Secondly assuming that functions \( h \) and \( \varphi \) do not depend on the variable \( t \), we prove the existence of an absorbing set in \( W^2_2(\Omega) \cap \mathcal{L}^{p+2}(\Omega) \). Also when the coefficients functions depend only on \( x \) as well as \( h \) and \( \varphi \), we prove some asymptotic regularity and the existence of global attractor in \( W^2_2(\Omega) \cap \mathcal{L}^{p+2}(\Omega) \).

**Decay estimates for unbounded solutions**

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It is well known that the solution of the heat equation with summable initial datum \( u_0 \) becomes “immediately bounded” and satisfies the decay estimate

\[
\|u(t, \cdot)\|_{L^\infty} \leq C \frac{\|u_0\|_{L^1}}{t^{\frac{n}{2}}}, \quad t > 0.
\]

This “strong regularizing effect” is not a peculiarity of the heat equation since it appears also for a lot of other parabolic problems, also nonlinear, degenerate or singular like the p-Laplacian equation, the porous medium equation, the fast diffusion equation, etc. In a recent paper we have proved that this phenomenon occurs when the solutions satisfy certain integral inequalities. We investigate here what happens when this regularizing effect does not appear and which is the solutions’ behavior in this case.
Bohmenian type boundary conditions for quantum hydrodynamics

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The quantum hydrodynamical model (QHD) coupled with the Poisson equation is able to describe the quantum effect appearing at nanoscale in many modern semiconductor devices. It can be obtained adding the Bohm potential to the classical hydrodynamical equations (HD) or directly from the Schroedinger equation. Here we derive a physically reasonable set of boundary conditions (BCs) for the QHD-Poisson system. We just consider the unipolar case, thus the holes concentration is neglected. These new BCs have two interesting explanations from the physical viewpoint. Firstly, if we consider the Bohm term as a correction for the pressure functional, it implies the conservation of the generalized enthalpy at the interface metal-semiconductor. Alternatively, assuming the Bohm term works together to the electrical potential, the BCs imply the equilibrium between diffusive and quantum forces. The existence and the uniqueness of a regular solution for the QHD-Poisson system is then discussed using these new BCs. The model is tested numerically on a toy device and the linear stability of the solution is discussed in a special case. The same consideration can be used to derive interface conditions between QHD and HD in the context of the hybrid models.

Localization of solutions of doubly nonlinear parabolic equations with anisotropic variable growth

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We present results on the properties of localization in time and space for solutions of doubly nonlinear parabolic equations with anisotropic and variable growth conditions. We derive the ranges of the variable exponents of nonlinearity where the solutions vanish or blowup in a finite time and study the asymptotic behavior of solutions for large time. Sufficient conditions of finite speed of propagation in space are established. We also study the effect of nonpropagation of disturbances from the data in certain directions due to the anisotropy of the diffusion operator. The results were obtained in collaboration with S.Antonsev.

REFERENCES


Global existence of the singularly perturbed Boussinesq-type equation

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We are concerned with the singularly perturbed Boussinesq-type equation including the singularly perturbed sixth-order Boussinesq equation, which describes the bi-directional propagation of small amplitude and long capillary-gravity waves on the surface of shallow water for bond number (surface tension parameter) less than but very close to 1/3. The existence and uniqueness of the global generalized solution and the global classical solution of the initial boundary value problem for the singularly perturbed Boussinesq-type equation are proved. The nonexistence of global solution of the above-problem is discussed and two examples are given.

Boundary layers to the Euler-Poisson equations for a multicomponent plasma

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In this talk, we study a boundary layer, called a sheath, which occurs on the surface of materials with which a multicomponent plasma contacts. For the sheath formation, the generalized Bohm criterion demands that the ions enter the sheath region with a high velocity. The motion of the multicomponent plasma is governed by the Euler-Poisson equations. The sheath is mathematically understood as the stationary solution to the equations. We show the unique existence and the asymptotic stability of the stationary solution under the the generalized Bohm criterion.

Decay structure of the regularity-loss type and the asymptotic stability for the Euler-Maxwell system

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In this talk, we study a Cauchy problem of the Euler-Maxwell system. The Euler-Maxwell system describes the dynamics of compressible electrons in plasma physics under the interaction of the magnetic and electric fields via the Lorentz force. Our purpose is to study the large-time behavior of solutions to the initial value problem for the Euler-Maxwell system in whole space. This system verifies the decay property of the regularity-loss type. Under smallness condition on the initial perturbation, we show that the solution to the problem exists globally in time and converges to the equilibrium state (and the stationary solution). Moreover we derive the corresponding convergence rate of the solutions. The key to the proof of our main theorems are to derive a priori estimates of solutions by using the energy method.
On the thin film approximation for the flow of a viscous incompressible fluid down an inclined plane

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Tatsuo Iguchi and Akinori Shiraiishi

We consider two-dimensional motion of liquid film of a viscous incompressible fluid down an inclined plane in the influence of the gravity and the surface tension. In order to investigate such a motion, a method of the thin film approximation is often used. It is the approximation by the perturbation expansion of the solution for the nondimensional parameter $\delta$ defined by ratio between the thickness of the liquid film and the typical wave length. In this study, we will give uniform estimates of the solution to the original Navier–Stokes equations in $\delta$ when the Reynolds number, the angle of inclination, and the initial date are sufficiently small.

Blow-up solutions in a Keller-Segel type system modelling the chemotaxis phenomenon

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M. Marras and G. Viglialoro

We study a Keller-Segel parabolic system, with time dependent coefficients. This system models a chemotaxis phenomenon. Sufficient conditions on data are introduced to obtain a lower bound for the blow-up time. This is a joint work with Monica Marras and Giuseppe Viglialoro.

Global existence and asymptotic behavior of solution for sixth order Boussinesq equation with damped term

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We investigate the small-data Cauchy problem for sixth order Boussinesq equation with damped term. We prove the global existence and asymptotic behavior of a solution. Our method and techniques rely upon contracting mapping principle, the dyadict decomposition and some properties of Bessel function.

Global existence and finite time blow up for a class of semilinear pseudo-parabolic equations

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In this paper, we study a class of semilinear pseudo-parabolic equations. By introducing a family of potential wells, we prove the invariance of some sets, global existence, nonexistence and asymptotic behavior of solutions with initial energy $J(u_0) \leq d$. Moreover, we obtain finite time blow up with high initial energy $J(u_0) > d$ by comparison principle.

Longtime dynamics for the strongly damped wave equation with critical and supercritical nonlinearities

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We are concerned with the longtime dynamics of the Kirchhoff equation with strong dissipation

$$u_{tt} - M(\|\nabla u\|^2)\Delta u - \Delta u_t + h(u) + g(u) = f(x).$$

We prove that the IBVP of the equation admits a unique global weak solution; the related dynamical system has a finite dimensional global attractor and an exponential attractor provided that $h(s)$ is quasi-monotone and both growth exponents $p$ and $q$ of the nonlinearities $h(s)$ and $g(s)$ are up to critical range, that is, $1 \leq p, q \leq p^* = \frac{N+2}{(N-2)^+}$. Moreover, the optimal regularity of the global attractor is established within further restrictions that the above-mentioned growth exponents are equal and up to subcritical range. In particular, when $M(s) = 1$, we further show that the IBVP of the equation possesses a global weak solution provided that the growth exponents $p$ and $q$ are fully supercritical, that is, $p = q > p^*$, and the related generalized semiflow has in natural energy space endowed with strong topology a global attractor.

The global existence of solution to the damped wave equations

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We consider the Cauchy problem for the system of weakly coupled semi-linear damped wave equations with small initial data. The global existence of the solution has been proved by constructing a contract mapping in some suitably function space in the supercritical case, which generalizes the results to the system in dimensional space $n \leq 5$. The proof is based on the estimates of solution operator for the linear damped wave equation.

Blow up in coupled parity-time symmetric nonlinear Schrödinger equations

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We consider the Cauchy problem for a system of two self-focusing nonlinear Schrödinger equations with linear and nonlinear coupling. The system is PT symmetric, which means that one of the equations includes the gain term while another equation accounts for the damping. We find sufficient conditions for the finite time blow up in the supercritical case (three or more spatial dimensions).
The proof is based on the virial technique arguments. Several other physically relevant particular cases are also discussed and illustrated numerically.