

**DESIGN OF EXPERIMENT FOR TUNING PARAMETERS OF  
AN ANT COLONY OPTIMIZATION METHOD FOR THE  
CONSTRAINED SHORTEST HAMILTONIAN PATH PROBLEM  
IN THE GRID NETWORKS**

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**ABSTRACT.** In a grid network, the nodes could be traversed either horizontally or vertically. The constrained shortest Hamiltonian path goes over the nodes between a source node and a destination node, and it is constrained to traverse some nodes at least once while others could be traversed several times. There are various applications of the problem, especially in routing problems. It is an NP-complete problem, and the well-known Bellman-Held-Karp algorithm could solve the shortest Hamiltonian circuit problem within  $O(2^n n^2)$  time complexity; however, the shortest Hamiltonian path problem is more complicated. So, a metaheuristic algorithm based on ant colony optimization is applied to obtain the optimal solution. The proposed method applies the rooted shortest path tree structure since in the optimal solution the paths between the restricted nodes are the shortest paths. Then, the shortest path tree is obtained by at most  $O(n^3)$  time complexity at any iteration and the ants begin to improve the solution and the optimal solution is constructed in a reasonable time. The algorithm is verified by some numerical examples and the ant colony parameters are tuned by design of experiment method, and the optimal setting for different size of networks are determined.

**1. Introduction.** There are many applications of the shortest Hamiltonian path (SHP) problem, e.g. travelling salesman problem [2, 3], routing problem with time windows [9], vehicle routing problem [8, 9], generalized travelling salesman problem [38, 41], warehouse management [33], etc. In a variant of the vehicle routing problem, there are some clusters of the customers and the capacitated vehicle should traverse the clusters to serve demands [38]. Vidal et al. [38] applied some preobtained SHPs between customers' clusters. Cao et al. [8] considered an open vehicle routing problem that the vehicle is not restricted to return to the source node, finally, and they considered the uncertain demands of the customers. Bula [7] modeled

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a hazardous vehicle routing problem as some Hamiltonian circuit with the same start and end nodes in a given depot node. Stetsyuk [35] considered a complete graph and he formulated the problem as a mixed-integer problem with at most  $2n^2$  variables and  $(n + 1)^2$  constrained.

Also, the grid topology of the networks is applied by vehicle routing, robotic, Geographic Information Systems (GIS) [40], generation of shortest test sequences for software testing [21], job scheduling, polar image segmentation, texture analysis, vehicle routing and military operations [4, 18, 24, 34]. Also, there are some applications in economic problems [16, 17, 19, 28, 29].

Ferone et al. [12] considered the constrained shortest path tour problem (CSPTP) and they proved it is an NP-complete problem. Andrade and Saraiva [13] studied the CSPTP, they formulated it as an integer linear programming and they applied an exact algorithm to solve the problem. Generally, the SHP problem is reduced to the CSPTP and it is an NP-complete problem [5, 12, 20].

Wang [39] improved an approximation algorithm based on four-point three-line inequality and the genetic algorithm that applies the local optimal Hamiltonian path to obtain some local optimal solutions. However, the parameters tuning of the hybrid genetic algorithm affected the convergence of the algorithm as well as the quality of the solution. The mutation probability is one of the important parameters for the diversity of the produced solutions. So, the big probability of the mutation causes not converged algorithm, also by the small probability, it will be easily trapped in a locally optimal solution [39]. Choong et al. [10] developed a bee colony algorithm for the traveling salesman problem (TSP), that every node should be visited exactly once and it returns to the source node, finally. Sudholt and Thyssen [36, 37] analyzed the running time of an ant colony optimization (ACO) method on some shortest path problems.

Yang et al. [41] proposed an ACO method to obtain fairly good solution for the generalized TSP, where tours are constructed by clusters instead of nodes; they applied the mutation operator of the genetic algorithm to avoid some local optimality, also they applied 2-OPT local search method to crossover and to reconnect two paths. Salari [32] proposed a hybrid ant colony algorithm with dynamic programming for the covering salesman problem, where a subset of nodes are restricted to visit and the other nodes should be located in a predetermined distance from the restricted nodes.

According to tune the ant colony parameters, the design of experiment method is applied to some instances and the optimal setting is determined. The tuning parameters of the metaheuristic algorithms were done experimentally, whereas the results were not confident optimal setting for different problems of different sizes. Mobin et al. [25] considered the simultaneous tuning of parameter in evolutionary algorithms; they prepared a full factorial design and determined the significant parameters of the evolutionary algorithms. However, their consideration was the evolutionary algorithms, e.g. genetic algorithms, simulated annealing, particle swarm optimization.

**2. The Constrained Shortest Hamiltonian Path Problem.** There are two given source and destination nodes, and some nodes are restricted to traverse at least once. So, in the optimal solution settings, the first traversed node is the source node and the last one is the destination node.

Let  $G = (A, N)$  be an undirected grid network with the arc set  $A$  and the node set  $N$ , then for any  $i \in N$ ,  $-l \leq i \leq l$ , the horizontal and orthogonal movements are possible as follows (see Figure1)

- $\begin{cases} (i+1, j) \in A, & \text{for } i = -l \text{ and } -l \leq j \leq l \\ (i, j+1) \in A, & \text{for } j = -l \text{ and } -l \leq i \leq l \end{cases}$
- $\begin{cases} (i-1, j) \in A, & \text{for } i = l \text{ and } -l \leq j \leq l \\ (i, j-1) \in A, & \text{for } j = l \text{ and } -l \leq i \leq l \end{cases}$
- $\begin{cases} (i+1, j), (i-1, j) \in A, & \text{for } -l < i < l \text{ and } -l \leq j \leq l \\ (i, j+1), (i, j-1) \in A, & \text{for } -l < j < l \text{ and } -l \leq i \leq l \end{cases}$

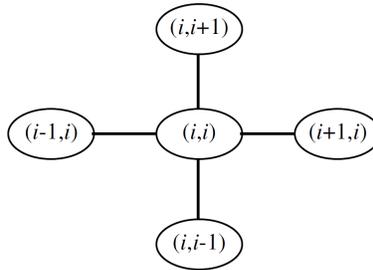


FIGURE 1. The horizontal and orthogonal movements in the grid networks

Obviously, there is a path between two arbitrary nodes in the grid networks, so a feasible solution could be obtained by the rooted shortest path tree.

The length of the rooted shortest path tree  $L_{tree}$  makes the upper bound  $2L_{tree}$  for the optimal constrained shortest Hamiltonian path (C-SHP). Let  $T_{sd}$  be the rooted shortest path tree in the source node  $s$ . The node set  $T'_{sd}$  contains the nodes along the shortest path from node  $s$  toward node  $d$ , and the other nodes located around the shortest path are involved in the node set  $T''_{sd}$  (see Figure 2). In Figure 2, the shortest path nodes are  $T'_{sd} = \{s, \dots, u, \dots, v, \dots, d\}$ , and the nodes around the shortest path are  $T''_{sd} = \{t_1, t_2, \dots, t_k\}$ , so the shortest path tree will be  $T_{sd} = T'_{sd} \cup T''_{sd}$ .

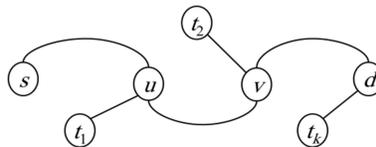


FIGURE 2. The shortest path tree

So, a feasible constrained Hamiltonian path could be constructed by traversing the arcs of  $T'_{sd}$  once and the arcs of  $T''_{sd}$  almost twice. Then, the optimal length of the C-SHP,  $L_{CSHP}^{Opt}$ , will be at most  $L_{T'_{sd}} + 2L_{T''_{sd}}$ , where  $L_{T'_{sd}}$  and  $L_{T''_{sd}}$  are the length of  $T'_{sd}$  and  $T''_{sd}$  paths, respectively. Finally, there is

$$L_{CSHP}^{Opt} \leq L_{T'_{sd}} + 2L_{T''_{sd}} \leq 2L_{tree}.$$

The restricted node set  $R$  contains the intermediate source and destination nodes and they are constrained to traverse at least once. Obviously, any optimal path between two arbitrary nodes  $u, v \in \{s, d\} \cup R$  should be the shortest path, however the Bellman principle [1] is not necessarily satisfied throughout all paths from the original source node  $s$  toward the original destination node  $d$ .

In the optimal C-SHP  $\bar{v}_0^* = s, \bar{v}_1^*, \dots, \bar{v}_{|R|}^*, \bar{v}_{|R|+1}^* = d$ , for  $\{\bar{v}_r^*, \bar{v}_{r+1}^*\}$  which is not constructed by the shortest path between  $\bar{v}_r^*$  and  $\bar{v}_{r+1}^*$ , then the length of the path  $\{\bar{v}_r^*, \bar{v}_{r+1}^*\}$  is greater than the shortest path and it is a contradiction with the optimal C-SHP. So, the length of the path between any successive nodes  $\bar{v}_r^*$  and  $\bar{v}_{r+1}^*$  should be the shortest path.

The C-SHP problem is an NP-complete problem which is reduced by 3SAT problem [27]. So, the problem is to find the optimal permutation of the nodes in  $R$  which has the minimum cost among all permutations.

**3. The Ant Colony Optimization Method.** The well-known Bellman-Held-Karp algorithm based on a dynamic programming method finds the shortest Hamiltonian path within  $O(2^n n^2)$  time complexity [5, 30]. So, some metaheuristic algorithms were developed to solve the problem in reasonable time complexity and acceptable quality of the solution.

Yang et al. [41] considered the ants were distributed over the nodes, randomly. There were memories for ants to avoid visiting the previously traversed nodes, and a tabu list of nodes is associated with each ant. The proposed algorithm by Salari et al. [32] starts with a set of feasible solutions, then the solutions were improved by some procedures, where the order of visited nodes are changed or replaced with other ones. De Santis et al. [33] considered the same start point for all ants and they proposed to update the pheromone trails for any best obtained solution in each iteration, instead of any movements by ants.

Initially, the movement directions are chosen randomly with equal probability by ants in the nodes of the set  $\{s, d\} \cup R$ . Then, they track the pheromone trails to obtain the shortest path toward any nodes of the set  $\{s, d\} \cup R$ . The pheromone depositions are updated for the obtained paths between two successive nodes by the ants.

Thus, some rooted shortest path trees are obtained by ants, and then the current solutions are improved in an iterative method. So, any iteration takes at most  $O(n^3)$  time complexity by a classic algorithms of the shortest path problem, for example, Dijkstra and Bellman-Fold algorithms. The proposed ACO algorithm for the C-SHP problem is presented in Figure 3.

The ACO parameters are set to determine the best solution according to the pheromone deposition and evaporation rates as the equations (1) and (2), respectively

$$\Delta\tau_{ij}^k(t) = \begin{cases} \frac{Q}{l_k(t)}, & \text{if arc } (i, j) \text{ is used by ant } k \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$\tau_{ij}(1+t) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^m \Delta\tau_{ij}^k(t) \quad (2)$$

Where,  $\Delta\tau_{ij}^k(t)$  is the additional pheromone by ant  $k$  on the path between two successive nodes  $i, j \in \{s, d\} \cup R$ , the other parameters are

- $l_k(t)$  is the traversed length by ant  $k$  for iteration  $t$
- $\rho$  is the evaporation rate

<p><b>Input</b> a grid network with some restricted nodes and the original source and destination nodes</p> <p><b>Initialize</b> obtain an initial feasible solution</p> <p><b>ACO parameters:</b> ant numbers <math>k</math> initial pheromone <math>\tau(0)</math> pheromone evaporation rate <math>\rho</math> parameters <math>\alpha</math>, <math>\beta</math> and <math>Q</math> iteration number <math>t</math></p> <p><b>While the iteration is satisfied do:</b> For ants 1 to <math>k</math> compute <math>l_k(t)</math>, <math>\Delta\tau_{ij}^k(t)</math> and <math>\tau_{ij}(t)</math> consider <math>\eta_{ij}(t)</math> and obtain <math>p_{ij}^k(t)</math> the next node is <math>j</math> probably</p> <p><b>Improvement</b> replace the worst paths with the best ones in the rooted tree</p>
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FIGURE 3. The ACO Algorithm for the C-SHP problem

- $m$  is the ant numbers using path  $\{i, j\}$
- $Q$  is set experimentally
- $\tau_{ij}(t)$  is the current pheromone trace on the arc  $(i, j)$
- $\tau_{ij}(t+1)$  is the pheromone trace on the arc  $(i, j)$  by update of  $\Delta\tau_{ij}^k(t)$  on the arcs.

So, the arcs with the largest amount of pheromone are considered to be in the best constrained Hamiltonian path.

Then, the transition probability from the current state to the next state shows the probability to traverse the arc  $(i, j)$  as the next decision, and it is computed by equation (3)

$$p_{ij}^k(t) = \frac{(\tau_{ij}(t))^\alpha (\eta_{ij}(t))^\beta}{\sum_{l \in N_i^k} (\tau_{il}(t))^\alpha (\eta_{il}(t))^\beta} \quad (3)$$

where  $\eta_{ij}(t)$  is the heuristic parameter and it is called the visibility of node  $j$  from node  $i$  that it is considered as  $1/d_{ij}$ ,  $d_{ij}$  is the shortest path length between the nodes  $i$  and  $j$  [41]; also  $\alpha$  and  $\beta$  are the parameters those are experimentally set by the user. The algorithm starts with a feasible solution that the order of nodes is randomly set and the shortest paths between the nodes are computed. Then, the worst obtained shortest path lengths in the current solution are replaced with the best ones in the obtained rooted trees.

**3.1. The ACO Algorithm Parameters.** The parameters of the ACO algorithm have been set experimentally [11], same as almost all of the other metaheuristic algorithms [32, 33]. Yang et al. [41] implemented an additional permutation parameter to avoid the local optimality. Salari et al. [32] in their hybrid ACO algorithm set the initial pheromone as  $1/(n * cost(T_0))$ , where  $cost(T_0)$  is the initial solution cost and the other parameters were  $\rho = 0.5$  for global pheromone update,  $\alpha = 1$ ,  $\beta = 3$ ,  $\eta_{ij}(t) = 1/c_{ij}$  for any  $(i, j) \in A$ , and  $\xi = 0.05$  for local pheromone updates of the traversed arcs by the ants. De Santis et al. [33] set the ACO parameters

$\rho = 0.5, 0.9$ ,  $\alpha = 1, 2$ ,  $\beta = 2, 5$ , it was set  $\alpha \leq \beta$ , the initial pheromone trail was  $\tau = 0.1$  or  $0.9$ , and the iteration number was 1000, 10000 and 20000, also it was set 2 for parameter  $Q$  and the number of ants  $m$  varied between 1 to 10.

However, we apply the design of experiment methods for tuning parameters of the ACO algorithm.

**3.2. The Initial Feasible Solution.** The initial solution could affect the convergence of the ACO algorithm. Here, the rooted shortest path tree is considered to obtain the initial feasible solution. However, some of the works let the algorithm start with an arbitrary random constructed initial solution [6,33]. Liu [23] compared three different types of the initial solution generators for the generalized travelling salesman problem: random generator, the nearest neighbor type I (NN1) and the nearest neighbor type II (NN2). We reformulate the methods presented by Liu [23] for the C-SHP problem. In the method NN1, two nearest neighbor nodes are inserted randomly between the source and destination nodes (see Figure 4a). In the method NN2, the nearest neighbors individually are located randomly between the source and destination nodes (see Figure 4b).

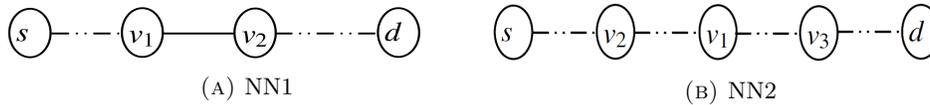


FIGURE 4. The initial neighborhood methods

Then, three methods are considered to construct the initial solution:

- the random order (RAN): the obtained nodes in the shortest path tree are located randomly between the source node  $s$  and the destination node  $d$
- the nearest neighbor type I (NN1): the paired successive nodes  $v_k$  and  $v_{k+1}$  in the shortest path tree are located randomly between  $s$  and  $d$  (Figure 4a)
- the nearest neighbor type II (NN2): the successive nodes  $v_k$  and  $v_{k+1}$  in the shortest path tree are individually located randomly between  $s$  and  $d$  (Figure 4b)

**4. The Design of Experiment Result.** The design of experiment (DOE) methods determines those parameters have a significant effect on the performance of the algorithm [22,31]. Also, they could rank the most important parameters [14]; then, the interactions between the parameters are computed. The parameters are called the factors in the DOE methods [26]. Ridge and Kudenko [31] considered 12 factors of ACO, however the initial solution was not considered. The lower and upper level values are considered with respect to Ridge and Kudenko [31], however they were implemented for the TSP. A factorial design [26] is applied as the screening phase to determine the most effective factors (parameters) and their linear or quadratic effects. Then, the response surface method and Box–Behnken design (BBD) [26] is implemented as the tuning phase [14,31].

So, there are two levels for the factors:  $\alpha$ ,  $\beta$ ,  $Q$ , the initial pheromone  $\tau(0)$ , the evaporation rate  $\rho$ , the initial solution method, and the instance size is considered as the block factor in the screening design phase. However, the axial points between the upper and lower levels are considered as Table 1; the levels' values are coded as -1, 0 and 1.

TABLE 1. The design factors and their levels

factors		levels		
		-1	0	1
A	$\alpha$	1	7	13
B	$\beta$	0	6	13
C	$Q$	1	2	4
D	$\tau(0)$	0.1	0.5	0.9
E	$\rho$	0.01	0.5	0.99
F	initial solution	RO	NN1	NN2
G	ant number	0.5	1	1.5
Blocks	instance size	small	moderate	large

The ants are nested in the restricted nodes; the instance sizes are considered as small, moderate and large, respectively for  $100 \times 100$ ,  $200 \times 200$  and  $400 \times 400$ , horizontal and vertical movements. The restricted nodes are randomly located and they are the proportion of the network size, 50, 100 and 200 for small, moderate and large networks, respectively. The iteration numbers are twice the sizes of the networks (200, 400, 800), and the local optimality is detected by the repetitions of the current solution as 10% of the iteration number.

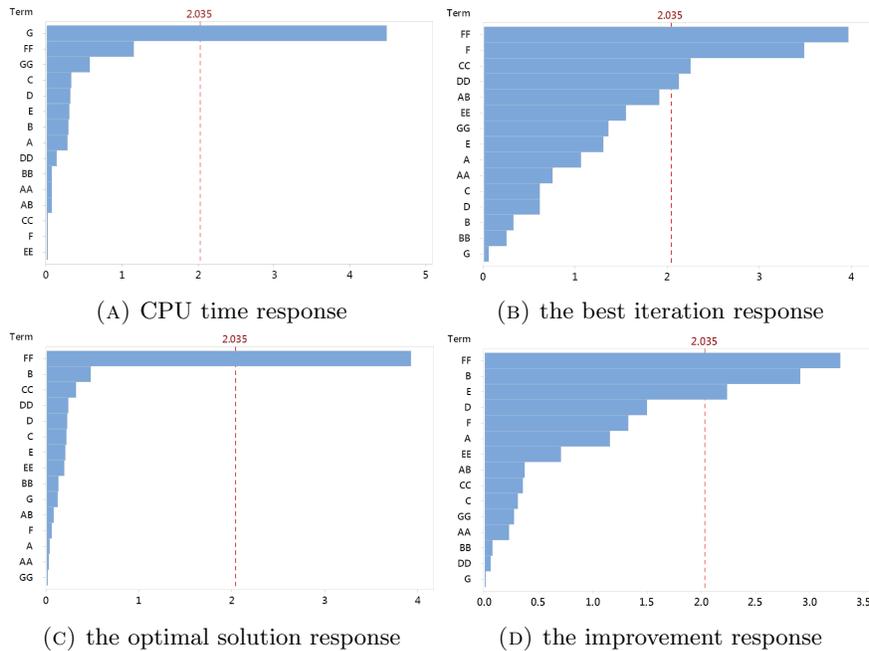


FIGURE 5. The pareto charts of the standardized effects for the responses in the screening phase for the network  $200 \times 200$

The confidence level is considered to be 95%, and the experiments are considered in three blocks with respect to the instance sizes. The screening phase of the considered parameters is done by Minitab software and it is designed as a definitive

screening design. The most effective factors are detected as the pareto charts in Figures 5. The terms are included both the linear and quadratics effects, then the most effective ones are detected as the ant number in CPU time (Figure 5a), the initial solution method,  $Q$ ,  $\tau(0)$  in the best iteration number, the initial solution method in the optimal solution, finally the initial solution method  $\beta$ ,  $\rho$  in the improvement of the initial solution.

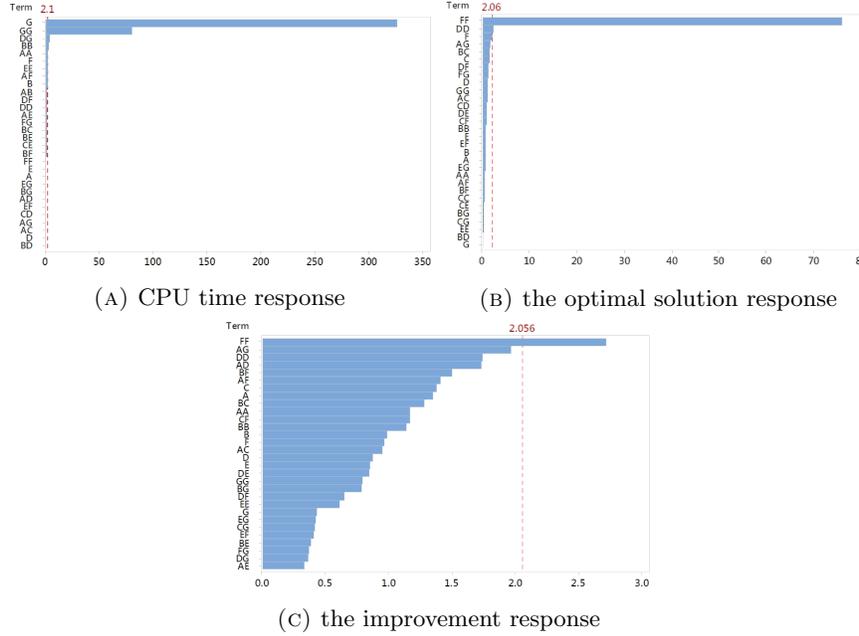


FIGURE 6. The pareto charts of the standardized effects in the network  $200 \times 200$  for the responses in Box–Behnken design

In the optimal tuning phase, a response surface method is applied for the grid networks of all sizes; however, the results are shown for the moderate size ( $200 \times 200$ ). After 62 runs for different settings of the parameters on any size of the network, BBD determines the most effective factors and the linear and quadratic factors' effects as shown in Figure 6. The quadratic effect of the initial solution factor is the most effective in the optimal solution and the improvement factors. The desirability method is applied to optimize the factor's levels with equally weighted responses by one (Figure 7).

The coded and uncoded optimal settings of the factors of the considered networks are obtained as Table 2. The optimal tuning of the parameters determines the optimal initial solution method is NN1, where paired nodes with shortest path length are located together. The iteration number was  $5 \times \text{network\_size}$  for the optimal tuning phase.

The prediction interval and the confidence interval of the optimal solutions of the networks are shown in Table 3. So, the implemented ACO algorithm on three size networks by optimal tuning of the parameters is obtained in Figure 8. By the restricted iteration numbers, the diagram of the optimal solution shows the ACO

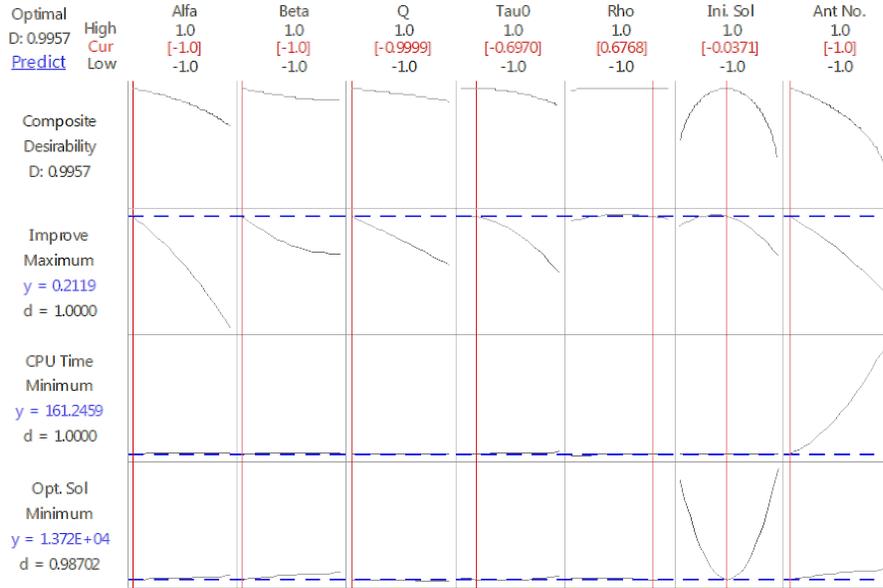


FIGURE 7. The optimized fitted response plots for the network 200×200

TABLE 2. The optimal coded and uncoded settings of the factors

network size	factors	$\alpha$	$\beta$	$Q$	$\tau(0)$	$\rho$	initial solution	ant number	desirability value
100×100	optimal coded	1	-1	1	-1	-0.68	0.05	-0.98	0.9640
	optimal uncoded	13	0	4	0.1	0.17	NN1	0.5	
200×200	optimal coded	-1	-1	-0.99	-0.70	0.68	-0.04	-1	0.9957
	optimal uncoded	1	0	1	0.22	0.83	NN1	0.5	
400×400	optimal coded	-1	-0.54	-1	1	1	0.15	-1	0.8996
	optimal uncoded	1	3	1	0.9	0.99	NN1	0.5	

algorithm continues to discover some better solutions where it is trapped in a locally optimal solution.

**5. Conclusion.** The constrained shortest Hamiltonian path problem in the grid networks was considered, where the movements at any node are possible in horizontal or vertical directions. A metaheuristic ant colony optimization algorithm was implemented for such an NP-complete problem. By design of experiments methods, two phases were developed to determine the important effects and to optimize the tuning parameters of the algorithm for different sizes of the grid networks. By factorial design, it is determined that the initial solution method is the most effective factor with the quadratic effect. Then, by the response surface method,

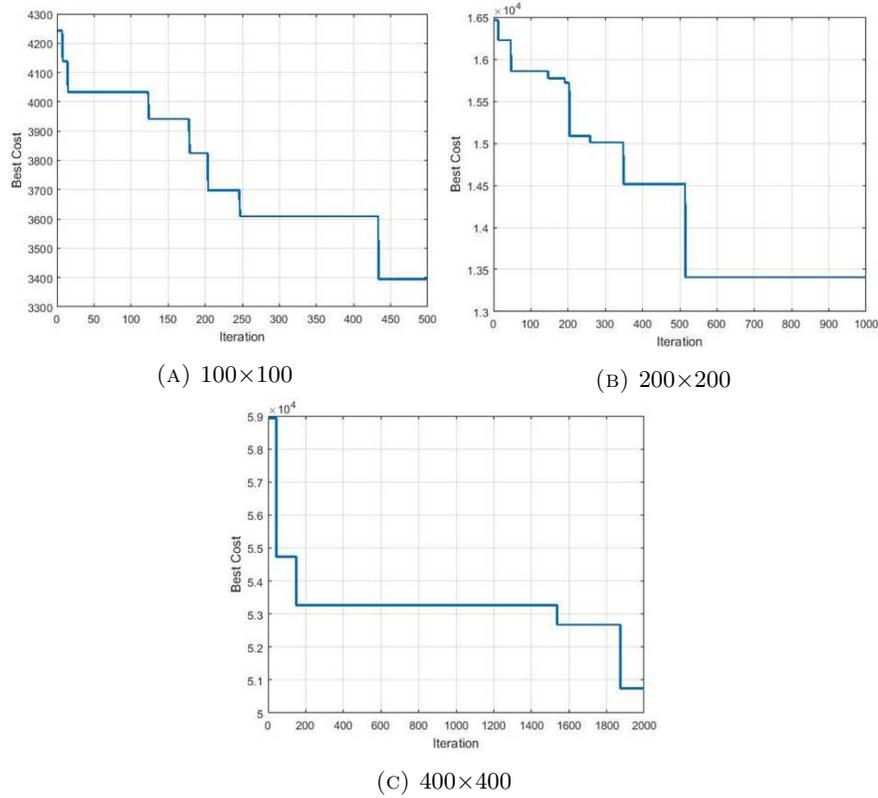


FIGURE 8. The solution diagram of ACO algorithm by optimal tuning of parameters

TABLE 3. The optimal prediction and the confidence intervals of the responses

Network	Response	Fit	SE Fit	95% CI	95% PI
$100 \times 100$	Improve	0.2970	0.0848	(0.1226, 0.4713)	(0.0829, 0.5111)
	CPU Time	18	10.1	(-2.8, 38.8)	(-7.6, 43.5)
	Opt. Sol.	3608	318	(2954, 4262)	(2804, 4412)
$200 \times 200$	Improve	0.2119	0.0534	(0.1020, 0.3217)	(0.0724, 0.3513)
	CPU Time	161.25	4.77	(151.44, 171.05)	(148.80, 173.69)
	Opt. Sol.	13719	654	(12375, 15063)	(12013, 15425)
$400 \times 400$	Improve	0.1605	0.0413	(0.0756, 0.2454)	(0.0555, 0.2655)
	CPU Time	1763	647	(434, 3092)	(119, 3407)
	Opt. Sol.	54087	1726	(50540, 57634)	(49699, 58474)

the optimal initial solution method NN1 is obtained by locating the paired nodes with shortest path length. Three responses the CPU time, the optimal solution and the improvement were considered in the optimization phase by Box–Behnken design. The confidence and the prediction intervals of three considered responses were determined for the networks.

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