

EVACUATION PLANNING BY EARLIEST ARRIVAL CONTRAFLOW

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ABSTRACT. The very challenging emergency issues because of large scale natural or man-created disasters promote the research on evacuation planning. The earliest arrival contraflow is an important model for evacuation planning that rescue as many evacuees as possible at any point in time by reversing the direction of arcs towards the safe destinations with increased outbound arc capacity. We present efficient algorithms to solve the earliest arrival contraflow problem on multiple sources and on multiple sinks networks separately. We also introduce an approximate-earliest arrival contraflow solution on multi-terminal networks.

1. Introduction. Evacuation planning is motivated by the challenging emergency issues in natural or man-created disasters like hurricanes, earthquakes, tsunamis, flooding, landslide, industrial and nuclear accidents, fire, terrorist attacks and wars. Well-known humanitarian relief incidents include the Haiti, Chile, Chichi, Bam and Kashmir earthquakes, the tsunami in the Indian Ocean and Japan, the major hurricanes Katrina, Rita and Sandy, and the September 11 attacks in New York City and Washington D.C. As a very recent example, Nepal Earthquake of magnitude 7.8 Richter scale that happened on 25 April 2015 and two powerful aftershocks killed at least 8891 people and injured 22302. It fully damaged 602567 private homes and 2687 government offices, and partially damaged 284479 private homes and 3776 government offices. After any kind of disasters, the problem of saving evacuees and normalizing the situation is challenging. Among many others, an effective evacuation solution has been lacked without a prior evacuation planning. A case study has been carried out before this disaster, but is still applicable [23]. Moreover, many scientists and geologists threatened for the biggest earthquake more than 8.4 on Richter scale in the far Western part of Nepal in future.

Normally, an evacuation planning problem is understood as the procedure of shifting maximum number of evacuees from disastrous zones to safety areas as quickly and efficiently as possible. Various mathematical models for flow maximization and time minimization have been studied in quite diversified research domains [6]. The former one sends the maximum flow to the safe areas in fixed time, however, the latter one shifts the given amount of evacuees in minimum time. Both

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objectives have been considered widely to solve different evacuation planning problems. However, an existence of integrated solution approach that could be accepted in wider range has been lacked. The evacuation planning is made on evacuation network.

The evacuation network is defined as a network that corresponds to a region to be evacuated in which the intersections of streets represent the nodes and the connections between these parts denote the edges. The initial locations of evacuees are the source nodes and the locations at safety regions are the sink nodes. Each arc has transit time and capacity. The group of evacuees that passes through the network over time is modeled as a flow.

Contraflow is a very useful model introduced in evacuation planning. It allows the arc reversals that increases the outbound road capacities. Through the network with increased capacity, contraflow problem shifts the maximum number of evacuees to the sinks and decreases the evacuation time as well. It seeks to remove traffic jams and makes the traffic systematic and smooth. It is emerging to react to different large scale natural and man-made disasters. However, it is a very challenging issue of finding a network reconfiguration with ideal lane directions satisfying the given constraints to optimize the given objective.

In literature, we find various mathematical models, heuristics, optimization and simulation techniques with contraflow for the transportation network, [6]. A formal definition of contraflow using graph and flow theory has been given in [16]. Authors in [15] have presented the first integer programming formulation. Moreover, they have presented two important heuristics, i.e., greedy and bottleneck relief, to solve the problem. They have shown that the evacuation time is reduced at least 40 percent with at most 30 percent of the total arc reversals. Moreover, they have proved that the contraflow problem is \mathcal{NP} -hard in general network. Thus, contraflow is a widely accepted model for a good solution rather than an optimal one for practical cases.

From the analytical point of view, we can find that the flow values obtained by contraflow models increase significantly that may be doubled for given time horizon. Moreover, the contraflow model is two times faster than the models without contraflow to transship the given value from the sources to the sinks. Some contraflow problems with efficient solution algorithms in particular networks have been solved in [24, 7, 20, 19, 21, 22].

In this work, we focus on the earliest arrival contraflow (EACF) problem. The estimation of evacuation time for shifting evacuees from the sources to the sinks is difficult. So, the problem of finding maximum flow from the beginning of time point is important in evacuation planning. In the dynamic contraflow evacuation networks, this kind of problem is considered as the EACF problem. The EACF problem does not need estimated time period in advance. In general, the EACF problem in multi-terminal network with unknown supplies and demands is unsolved. Moreover, for the fixed supplies and demands, there is no polynomial algorithm to solve not only the EACF problem but also the earliest arrival flow (EAF) problem without contraflow on multi-terminal networks. However, we introduce the EACF with fixed supplies and demands, and present efficient algorithms to solve the problem on multi-source or multi-sink networks.

First, we solve the EACF problem on multi-source and a single sink networks having arbitrary transit time on each arc. Then, taking zero transit time on each arc, we solve it on multi-sink networks. If the evacuation scenarios include a ship

with several life boat or widespread pick-up bus in an urban evacuation, then naturally the multiple sinks networks occur. In zero transit time, arc capacities of networks restrict the quantity of flow that can be sent at any one time with arc reversal capability. We also introduce an approximate EACF problem and solve it on multi-terminal networks with both arbitrary transit time and zero transit time. Notice that the earliest arrival flow in networks with multiple sources and/or sinks associated with supplies/demands, where we have to transship the supplies within given time period are sometimes also called earliest arrival transshipment but here we write the easier term earliest arrival flow.

The paper is organized as follows. In Section 2, we model the contraflow evacuation planning problem. The overview of contraflow evacuation planning problem is mentioned in Section 3. In section 4, we present our main results on the earliest arrival contraflow problem on multiple sources, on multiple sinks and on multi-terminal networks. Section 5 concludes the paper.

2. Basic model. The disastrous area is represented by a directed graph $G = (V, A)$, $|V| = n$ and $|A| = m$, with a set of nodes V and a set of arcs A . As the case is of lane reversal scenarios, two way network configuration is allowed. Let $S \subset V$ and $D \subset V$ be a set of source nodes where evacuees are initially located and a set of sink nodes with enough capacity. Nodes s and d represent the single-source and single-sink. We represent the maximum units of flow that may enter the initial node of arc e per time period by its capacity $b_A(e)$ and the time needed to travel one unit of flow on the arc $e = (v, w)$ from node v to w by transit time $\tau(e)$. Each node in V has node capacity $b_V(v)$ that represents the amount of flow allowed to hold at node v . The vectors $\mu(s)$ and $\nu(d)$ represent the given supply and demand at each source and sink, respectively. We assume that $A_d^{out} = A_s^{in} = \emptyset$, where $A_v^{out} = \{(v, w) \in A\}$ and $A_v^{in} = \{(w, v) \in A\}$ for the node $v \in V$.

The transportation network $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d), T)$ is represented by the collection of all data in the evacuation scenario with predetermine time T . We assume a finite time horizon T that means everything must happen before time T . Time can increase in discrete increments or continuously. We consider the discrete time with a suitable time unit like at times $t = 0, 1, \dots, T$ and all time related parameters are integers. The choice of time unit effects the problem directly i.e., if the time unit is shorter then the problem is more complex. Let \mathbf{T} be the domain of time i.e., $\mathbf{T} = \{0, 1, \dots, T\}$.

Let the reversal of an arc $e = (v, w)$ be $e^{-1} = (w, v)$. For a contraflow configuration of a network \mathcal{N} with symmetric travel times, the auxiliary network $\bar{\mathcal{N}} = (V, E, b_E, b_V, \tau, S, D, T)$ consists of the modified arc capacities and travel times as

$$b_E(\bar{e}) = b_A(e) + b_A(e^{-1}), \text{ and } \tau(\bar{e}) = \begin{cases} \tau(e) & \text{if } e \in A \\ \tau(e^{-1}) & \text{otherwise} \end{cases}$$

where, an edge $\bar{e} \in E$ in $\bar{\mathcal{N}}$ if $e \vee e^{-1} \in A$ in \mathcal{N} . The remaining graph structure and data are unaltered.

The nonnegative functions x_{dyna} and x_{stat} define the dynamic and static network flows on $A \times \mathbf{T}$ and A , respectively. Let $\mathcal{N}_{x_{stat}}^R = (V, \vec{A} \cup \overleftarrow{A})$ be the residual network of \mathcal{N} where $\vec{A} = \{\vec{e} = e \mid x_{stat}(e) < b_A(e)\}$ with capacity $b_A(e) - x_{stat}(e)$ and transit time $\tau(e)$, and $\overleftarrow{A} = \{\overleftarrow{e} = (\text{head}(e), \text{tail}(e)) \mid x_{stat}(e) > 0\}$ with capacity $x_{stat}(e)$ and a transit time $-\tau(e)$.

A dynamic s - d flow x_{dyna} for given time T with arc reversal capability satisfies the *flow conservation* and *capacity constraints* (1-3). The inequality flow conservation constraints allow to wait flow at intermediate nodes, however, the equality flow conservation constraints force that flow entering an intermediate node must leave it again immediately.

$$\sum_{\sigma=\tau(e)}^T \sum_{e \in A_v^{in}} x_{dyna}(e, \sigma - \tau(e)) - \sum_{\sigma=0}^T \sum_{e \in A_v^{out}} x_{dyna}(e, \sigma) = 0, \forall v \notin \{s, d\} \quad (1)$$

$$\sum_{\sigma=\tau(e)}^t \sum_{e \in A_v^{in}} x_{dyna}(e, \sigma - \tau(e)) - \sum_{\sigma=0}^t \sum_{e \in A_v^{out}} x_{dyna}(e, \sigma) \geq 0, \forall v \notin \{s, d\}, t \in \mathbf{T} \quad (2)$$

$$0 \leq x_{dyna}(e, t) \leq b_A(e, t), \quad \forall e \in A, t \in \mathbf{T} \quad (3)$$

The *earliest arrival flow (EAF) problem* with arc reversal capability maximizes the $val(x_{dyna}, t)$ in (4) for all $t \in \mathbf{T}$ satisfying the constraints (1-3). We denote the maximum flow value by $val_{max}(x_{dyna}, t)$.

$$val(x_{dyna}, t) = \sum_{\sigma=0}^t \sum_{e \in A_s^{out}} x_{dyna}(e, \sigma) = \sum_{\sigma=\tau(e)}^t \sum_{e \in A_d^{in}} x_{dyna}(e, \sigma - \tau(e)) \quad (4)$$

For a network \mathcal{N} , the time expanded network $\mathcal{N}(T) = (V_T, A_M \cup A_H)$ is defined as an expansion of the dynamic network where each node v of the static graph is copied T times to obtain a node $v(t)$ for each $v \in V$ and each $t \in \{0, \dots, T\}$. For each arc $e = (v, w) \in A$, the arc from $v(t)$ to $w(t + \tau(v, w))$ has capacity $b_A(v, w)$, called movement arc. For each arc $e = (v, w) \in A$, the arc from $v(t)$ to $v(t + 1)$ has capacity $b_V(v)$, called holdover arc which allows storage at the node.

Let \mathcal{N}^* be the two-terminal extended network of multi-terminal network \mathcal{N} obtained by adding a super-terminal node (\star) and introducing arcs (\star, s_i) to each $s_i \in S$ with infinite capacity and zero transit time, and arcs (d_i, \star) to each $d_i \in D$ with infinite capacity and transit time $-(T + 1)$ for given time period T .

3. Evacuation planning by contraflow. Authors in [30] focused that planners should have guidelines for the design, operation and location of contraflow segments. Most of the published reports of transportation departments dealt the operational and managerial aspects of contraflow such as merging, signal control and cost in which they designed the network configuration depending on empirical guesses. Two evacuation planning algorithms: all-links and fastest-links, using contraflow techniques to support a smart traffic evacuation management system (STEMS) had been presented in [12] by ignoring the overall capacity of road. After hurricanes Katrina and Rita, author in [17] identified the planning problems and criticized the unplanned contraflow ordered and failure to use contraflow lanes. Moreover, author suggested a significant time saving possibilities with contraflow techniques.

Authors in [27] presented a mesoscopic model for contraflow networks in which they reverse the capacity using a mathematical optimization framework. Moreover, they presented an iterative tabu-based heuristic approach to address the proposed capacity reversibility optimization in [28]. With a formal definition of contraflow using graph and flow theory, two heuristics: flip high flow edge and simulated annealing have been presented in [16] to minimize the evacuation time with contraflow approach. Authors in [15] studied a macroscopic model integrating road capacity

constraints, multiple sources, congestion and scalability, and presented its computational difficulties.

In this section, we summarize some contraflow problems that are analytically solved. The two terminal maximum static contraflow (MSCF) problem has been solved by establishing the fact that a MSCF is equivalent to a maximum static flow (MSF) on corresponding auxiliary network, [2, 24]. It requires $O(h_1(n, m) + h_2(n, m))$ time, where $h_1(n, m) = O(n^2\sqrt{m})$, [5] and $h_2(n, m) = O(nm)$, [1] denote the time required to solve the MSF problem and the flow decomposition, respectively. To solve the general MSF problem, a prior reduction of multi-terminal network \mathcal{N} into the s_0 - d_0 MSF problem with super-source s_0 connecting to each $s \in S$ having arc capacities equal their respective surplus and connecting each $d \in D$ to super-sink d_0 having arc capacities equal their respective deficits is required.

Authors in [2, 24] solved a MDCF problem on two-terminal network \mathcal{N} by solving a s - d MDF problem in the auxiliary network $\bar{\mathcal{N}}$. Their algorithm for s - d MDCF problem determines temporally repeated dynamic flow in $\bar{\mathcal{N}}$ which is optimal over all dynamic flows in it. As a procedure a minimum cost flow (MCF) solution has been obtained and the flow has been decomposed into paths and removable cycles. An arc $(w, v) \in A$ is reversed if and only if the flow on (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along $(v, w) \notin A$. This problem is solved in time $O(h_2(n, m) + h_3(n, m))$, where $h_2(n, m) = O(nm)$ and $h_3(n, m) = O(n^2m^3 \log n)$, [10] are the time required to solve the flow decomposition and the MSF problem, respectively. The MDCF problem on multi-terminal network is NP-hard in the strong sense even with two sources and one sink or vice versa, see also Example 1. The proofs follow by reductions from the problems 3-SAT and PARTITION, [15, 24].

Example 1. [2, 24] There is no feasible flow within time $T = 6$ using only one of the arcs (x, y) or (y, x) in Figure 1. At time 1, one would switch (y, x) in order to increase the capacity and at time 3, one would switch it back again to achieve a feasible flow within a shorter time. Hence, the possibility of using both arcs leads to the problem NP-Complete.

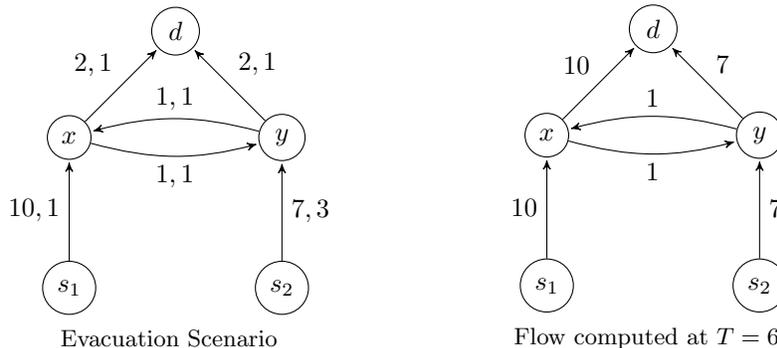


FIGURE 1. Two sources and single sink MDCF scenario and its solution

The quickest contraflow (QCF) problem on two-terminal network is polynomially solvable. Its solution is based on parametric search algorithms of [4]. The multi-terminal QCF problems are harder than 3-SAT and PARTITION [2, 24].

With arc reversals only once at time zero, an optimal solution to the s - d MDCF (and s - d EACF) problem on a two-terminal series-parallel graph has been obtained in [7, 20]. The algorithm for this problem has been obtained by a modification of MDCF algorithm in [2, 24] using the MCCF algorithm of [25]. The main advantage in series-parallel graphs is that every cycle in the residual network has nonnegative cycle length. This solves the MCCF problem introduced in [9] for the MDF problem in the auxiliary network $\bar{\mathcal{N}}$. The temporally repeated flow thus obtained is an optimal solution to the s - d EACF problem on a two-terminal series-parallel graph in time $O(nm + m \log m)$.

Pyakurel and Dhamala [22] show that the EACF problem for general graphs always exists with relaxation of arc reversal capability at a number of times when an EAF solution demands this property. This solution is based on the algorithm in [2, 24] for the MDCF problem and algorithms in [29, 18] for EAF problem. The solution can be obtained in polynomial time in the size of time expanded network. Thus, its complexity is pseudo-polynomial time.

The lex-max static contraflow problem on multi-terminal static network \mathcal{N} has been solved with polynomial algorithm in [21, 22]. Their algorithm modifies the MSCF algorithm of [2, 24] using lex-max static flow algorithm of [18]. For the multi-terminal dynamic network \mathcal{N} , the lex-max dynamic contraflow algorithm has been developed that solves the lex-max dynamic contraflow problem polynomially. This algorithm is based on the the MDCF algorithm of [2, 24], and the lex-max dynamic flow algorithm of [13].

4. Earliest arrival contraflow on multi-terminal networks. The more restricted multi-source EACF problem with arc reversals allowed only at zero time must be NP-hard as the MDCF problem is NP-hard. In this section, we introduce the earliest arrival transshipment contraflow (EATCF) problem. Let $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d))$ be an evacuation network with multiple sources S and sinks D i.e., multi-terminal. Moreover, each source and sink have a supply $\mu(s)$ for all $s \in S$ and demand $\nu(d)$ for all $d \in D$, respectively. Now, the EACF problem is to send the supplies from the sources to the sinks so that the amount of flow received at the sinks by time t is maximum possible, for all $t > 0$ by reversing the direction of arcs whenever it necessary. If time horizon T is given within which all supplies should be shifted to the sinks from the sources with a feasible dynamic flow x_{dyna} , then the problem turns into the EATCF problem and its network is $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d), T)$. However, for the sake of convenience we use the terminology EACF problem for the EATCF problem.

Problem 1. *Given a network $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d), T)$ with integer inputs, the EACF problem is to find a feasible dynamic flow from the sources S to the sinks D that is maximal for all time periods $0 \leq t \leq T$ satisfying given supplies and demands if the direction of arcs can be reversed whenever it requires without any processing cost.*

For multi-source and a single sink evacuation networks, we establish the solution strategy of the EACF problem in Subsection 4.1. We categorize the multi-sink networks with a single source that allow the EACF solution and present efficient algorithms to solve the problem in Subsection 4.2. Moreover, we propose value-approximate algorithms to obtain approximate EACF solution on multi-terminal networks (cf. Subsection 4.3).

4.1. Earliest arrival contraflow on multi-source. We consider the evacuation network $\mathcal{N} = (V, A, b_A, \tau, S, d, \mu(s), \nu(d), T)$ with multi-source S and a single sink d in which each source and sink have respective supply and demand. In this section, we introduce the multi-source EACF problem and present an efficient algorithm to solve the problem, Problem 1 on the network $\mathcal{N} = (V, A, b_A, \tau, S, d, \mu(s), \nu(d), T)$. Our algorithm is obtained by a combination of MDCF algorithm presented in [2, 24] for the MDCF problem and the algorithm developed in [3] for the EAF problem on multi-source network with given supplies and demands.

Algorithm 1. Earliest arrival contraflow with fixed supply and demand

1. Given, multiple sources network $\mathcal{N} = (V, A, b_A, \tau, S, d, \mu(s), \nu(d), T)$ with integer inputs
2. Construct the auxiliary network $\bar{\mathcal{N}} = (V, E, b_E, \tau, S, d, \mu(s), \nu(d), T)$ of \mathcal{N}
3. Construct the extended network $\bar{\mathcal{N}}^*$ of $\bar{\mathcal{N}}$
4. Compute the earliest arrival pattern $p(t)$ on $\bar{\mathcal{N}}^*$ by using the algorithm of [3]
5. Convert the $p(t)$ from Step 4 into an EAF according to [3]
6. Arc $(w, v) \in A$ is reversed, if and only if the flow along arc (v, w) is greater than $b_A(v, w)$ or if there is a non-negative flow along arc $(v, w) \notin A$ and the resulting flow is an EAF on $\bar{\mathcal{N}}$.
7. Obtain EACF solution on \mathcal{N} .

In order to solve the multi-source EACF problem on multi-source evacuation network $\mathcal{N} = (V, A, b_A, \tau, S, d, \mu(s), \nu(d), T)$, we first construct the corresponding auxiliary network $\bar{\mathcal{N}} = (V, E, b_E, \tau, S, d, \mu(s), \nu(d), T)$ according to the Step 2 of Algorithm 1. Recall that auxiliary network is constructed by contraflow configuration. The multi-source auxiliary network $\bar{\mathcal{N}} = (V, E, b_E, \tau, S, d, \mu(s), \nu(d), T)$ is converted into a single source network by constructing extended auxiliary network as follows. We add a super-terminal node s_0 which is connected to each source $s \in S$ by uncapacitated arcs (s_0, s) with zero transit time, and that can be reached from the sink d by an uncapacitated dummy arc (d, s_0) . On the extended auxiliary network $\bar{\mathcal{N}}^*$ thus constructed, all nodes in S become intermediate nodes and their entire supplies $\mu(S)$ are shifted to the super-source s_0 . Thus, the extended auxiliary network $\bar{\mathcal{N}}^*$ in Step 3 is two terminal network. Now we solve the static minimum cost circulation in $\bar{\mathcal{N}}^*$ where the transit times on arcs are interpreted as cost coefficients.

Then, we obtain a feasible dynamic flow from the source s_0 to the sink d by computing the minimum cost circulation flow as in [3] on extended auxiliary network $\bar{\mathcal{N}}^*$ that induces a dynamic flow on auxiliary network $\bar{\mathcal{N}}$ where $\mu(S)$ units of flow are being sent from the sources in S to the sink d within time horizon T . But the individual supplies at the source nodes might be violated by the induced dynamic flow on $\bar{\mathcal{N}}$. To overcome from this difficulty, an earliest arrival flow pattern $p(t)$ is defined on $\bar{\mathcal{N}}^*$.

The earliest arrival pattern $p(t)$ on $\bar{\mathcal{N}}^*$ is the maximum flow $val_S(x_{dyna}, t)$. For every $t \geq 0$ it holds that $p(t) \leq val_S(x_{dyna}, t)$. If $p(t) = val_S(x_{dyna}, t)$, for all $t \geq 0$, we are done. Otherwise we have to prove $p(t) = val_S(x_{dyna}, t)$. We obtain the earliest arrival pattern $p(t)$ using the algorithm of [3] on $\bar{\mathcal{N}}^*$ to avoid the violation of individual supplies at the source nodes.

Lemma 4.1. [3] *The earliest arrival pattern p can be computed and implemented in strongly polynomial time in the input plus output size.*

Authors in [3] obtained the piece-wise linear earliest arrival pattern p in time polynomially in the input size plus the number of breakpoints. We adapt their result of continuous model in discrete model using the natural transformation of [8].

The obtained earliest arrival pattern is now turned into an EAF. The problem of finding an EAF solution can be reduced to finding a transshipment dynamic flow in $\overline{\mathcal{N}}^*$ with k additional arcs leading from d to k new sink nodes d_1, \dots, d_k where the sink d will be an intermediate node. An EAF in $\overline{\mathcal{N}}$ with earliest arrival pattern $p(t)$ naturally induces a feasible transshipment dynamic flow with given time horizon satisfying all supplies and demands on $\overline{\mathcal{N}}^*$. The induced transshipment dynamic flow on $\overline{\mathcal{N}}^*$ naturally gives an EAF solution in $\overline{\mathcal{N}}$. The transshipment dynamic flow on $\overline{\mathcal{N}}^*$ has been obtained by using the algorithm of [13].

Lemma 4.2. [3] *Given the earliest arrival pattern p with k breakpoints for the network $\overline{\mathcal{N}}$, an EAF solution in $\overline{\mathcal{N}}$ can be obtained by computing a transshipment dynamic flow in $\overline{\mathcal{N}}^*$ with k additional nodes and arcs.*

The running time of transshipment dynamic flow algorithm [13] is bounded by a polynomial in the encoding size of the input $\overline{\mathcal{N}}^*$. The complexity of $\overline{\mathcal{N}}^*$ is similar as the complexity of $\overline{\mathcal{N}}$ which is the encoding size of $\overline{\mathcal{N}}$ plus the encoding size of p [3]. Thus the EAF solution on multi-sources network $\overline{\mathcal{N}}$ is obtained in time polynomial in the input plus output size.

Theorem 4.3. *Algorithm 1 computes the EACF solution optimally.*

Proof. First, we show that any solution of the Algorithm 1 is feasible for the network \mathcal{N} . According to [2, 24], Step 2 and Step 6 are feasible. The feasibility of Steps (3-5) are verified by using algorithm of [3].

Next, we prove the optimality of the solution induced by the algorithm. Note that any optimal solution to an EAF problem with arc reversal on multiple sources network \mathcal{N} is also a feasible solution to the EAF problem on the auxiliary network $\overline{\mathcal{N}}$. Lemma 4.2 induced the EAF-solution on $\overline{\mathcal{N}}$. As the amount of flow sent from sources S to a sink d induced from Steps 3-5 is not changed in Step 6, an optimal solution to the EACF problem on \mathcal{N} is obtained. \square

Theorem 4.4. *Algorithm 1 solves the EACF problem in polynomial time complexity.*

Proof. Step 2, Step 3 and Step 6 of the Algorithm 1 can be computed on linear time. The running time of solving the EAF problem on $\overline{\mathcal{N}}$ in Steps (4-5) is polynomial in the input plus output size. This follows the overall running time of Algorithm 1. \square

However, the EACF problem on network with the multi-sink having arbitrary transit time do not necessary exists. We explain it with a counter Example 2.

Example 2. All arcs have unit capacity and the transit time as shown in Figure 2(a). The contraflow configuration of Figure 2(a) is as shown in Figure 2(b). Flow 4 units at the source can satisfy the demands at the sinks by time 3 if we send the flow to the sink d_1 first, (c.f., Figure 2(c)), otherwise it will be achieved at time 2 as shown in Figure 2(d). This shows that there is no EACF solution on networks with multiple sinks.

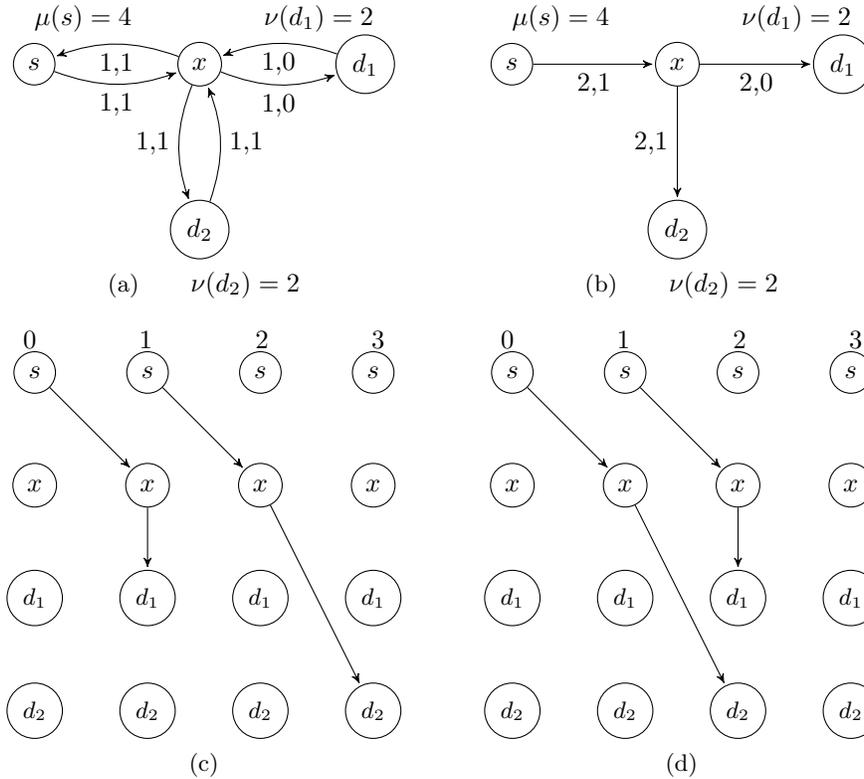


FIGURE 2. Non-existence of earliest arrival contraflow on multiple sinks

4.2. **Earliest arrival contraflow on multiple sinks.** First, we consider the EACF problem, (cf. Problem 1) on network $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d), T)$ with multi-sink. As the problem is not solvable with arbitrary transit time (cf. Example 2) on multi-sink network, we consider a special case of zero transit time on each arc of the network. But even in the special case, the EACF problem may not be solved because some sources may send flow in wrong sinks as explain in Example 3.

Example 3. Consider an evacuation network as shown in Figure 3(a) with two sources and two sinks in which each arc has unit capacity and zero transit time, and nodes have given supplies and demands. The contraflow configuration of the network is as shown in Figure 3(b). At first time step, we can send two units of flow on each arc as shown in Figure 3(c). This is the best possibility from which we can shift only six units of flow. At time step two also, we can send only six units of flow. If we leave the arc (s_2, d_1) empty and send the flow, we can send all flows i.e., eight units in two steps as shown in Figure 3(d). Since we can send either six units of flow in first time step or eight units in the first and second time steps, no EACF exists in the network.

Depending upon Example 3, we conclude that even in the networks with two sources and two sinks, and zero transit time on each arc, the EACF solution does not exist. Now, we consider the network with a single source s and multi-sink D for Problem 1. The existence of the EAF in networks with a single source and

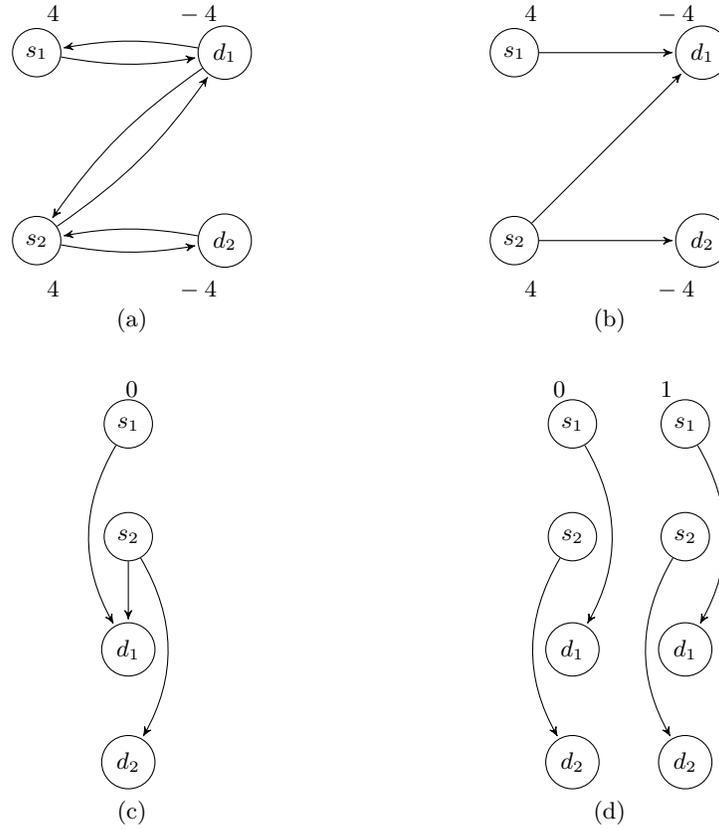


FIGURE 3. EACF solution does not always exist on multiple terminal networks

multi-sink having zero transit time has been proved in [26]. Depending on these results, we present, in this section, the solution procedures for the EACF problem on network $\mathcal{N} = (V, A, b_A, \tau, s, D, \mu(s), \nu(d), T)$ with a single source and multi-sink having zero transit time on each arc.

We investigate different networks with multi-sink where the existence of EACF solution is possible. For each multi-sink network \mathcal{N} with graph $G = (V, A)$, we solve the EACF problem in the corresponding auxiliary networks $\bar{\mathcal{N}}$ with graph $G = (V, E)$. The auxiliary network $\bar{\mathcal{N}} = (V, E, b_E, \tau, s, D, \mu(s), \nu(d), T)$ is constructed by reversing the arcs without any processing cost and the reversed arc capacity is increased but the transit time remains zero. An arc $(w, v) \in A$ is reversed if and only if the flow on (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along $(v, w) \notin A$ and the resulting flow is the EAF with arc reversals capability.

For a graph G , an in-tree (out-tree) with root $w \in V$ is a directed path such that there is exactly one directed path from v to w (from w to v) for every node $v \in V$. The number of arcs contained in the longest path gives the length of an in-or-out tree. Thus, if the network \mathcal{N} is restricted to tree network with length of in-or-out tree ≤ 2 , then there exists an EACF for satisfying the capacity constraint and supplies/demands. We solve the EACF problem on the tree network \mathcal{N} by solving the EAF problem as in [26] on the corresponding auxiliary network $\bar{\mathcal{N}}$.

Standing on the result of [26], we propose that for the multi-sink network $\mathcal{N} = (V, A, b_A, \tau, s, D, \mu(s), \nu(d), T)$ with graph G and transit time $\tau(e) = 0$ for all $e \in A$, an EACF exists if and only if G contains no sub-graphs G_1 and G_2 satisfying all capacities constraints and supply/demand. Thus, we state the Theorem 4.5.

Theorem 4.5. *For a network \mathcal{N} with graph $G = (V, A)$, EACF with zero transit time do always exist with arc reversal capability for each of the following graph construction.*

1. G with single length path
2. G with path of length two
3. G that contains two paths starting in the same node but continuing disjointly
4. G that contains two paths of length two that start in different nodes but end in the same node
5. G that contains at least one path of length two

On proof of Theorem 4.5, we construct the corresponding auxiliary network $\overline{\mathcal{N}} = (V, E, b_E, \tau, s, D, \mu(s), \nu(d), T)$ with graph $G = (V, E)$ having arc capacity b_E and zero transit time of given network \mathcal{N} . On the auxiliary network $\overline{\mathcal{N}}$, we prove the existence of the EAF solution according to [26].

If the graph G has paths with single length, there is no possibility of creating sub-graphs of G as shown in Figure 4(a) where node s have two outgoing arcs but it does not contain any incoming arc. As a result, the network $\overline{\mathcal{N}}$ with graph $G = (V, E)$ as in Case 1 allows an EAF. Similar result will be obtained on a network $\overline{\mathcal{N}}$ with graph as in Case 2 in which the graph G contains a path of length two that is either parallel to another path of length two or to one of length one as shown in Figure 4(b).

For a network $\overline{\mathcal{N}}$ with graph G as in Case 3, let (s, x) and (x, d_2) be the arcs of the first path and (s, y) and (y, d_1) be the arcs of the second path as shown in Figure 4(c). Then, the graph G is an out-tree of length two. The root of out-tree is s which does not have any incoming arcs. It has only two outgoing arcs and their continuation disjointly. Moreover, we know that every out-tree with length ≤ 2 allows an EAF with zero transit time and there is no sub-graph of G . Thus, we conclude the result that an EACF always exists on graph in Case 3 of Theorem 4.5.

If the graph $G = (V, E)$ in Case 4 on network $\overline{\mathcal{N}}$ contains two paths of length two that start in different nodes, but end in the same node, then there always exists an EAF. It can be solved as in Subsection 4.1. But if $G = (V, E)$ contains two paths of length two start in same node but end in different nodes as shown in Figure 4(d), then we define another $\overline{\mathcal{N}}_r$ with graph $G_r = (V, E_r)$ where $E_r = \{(w, v) \mid (v, w) \in E\}$, capacities $b_{E_r}^r = b_E$, integer supply $\mu_r(v) = -\mu(v)$ for $v \in V$. Then, an EAF can be computed in $\overline{\mathcal{N}}$ if and only if $\overline{\mathcal{N}}_r$ allows a solution for an EAF [26]. As the network $\overline{\mathcal{N}}_r$ acts as multi-source-single sink networks, the EAF problem is easily solved as in Subsection 4.1. The EAF solution on $\overline{\mathcal{N}}$ is equivalent to the EACF on \mathcal{N} .

The graph G in Case 5 looks different than other four types in which it contains at least one path of length 2. Starting with this path, we search the additional nodes and arcs that can be the part of graph G . Then, the shape of graph may be star-shaped or ordered trees. In both cases, an EAF problem can be obtained in the corresponding auxiliary network $\overline{\mathcal{N}}$ including the graph thus formed as in [26]. That gives the EACF solution with zero transit time satisfying all capacity constraints and supply/demand in original network \mathcal{N} .

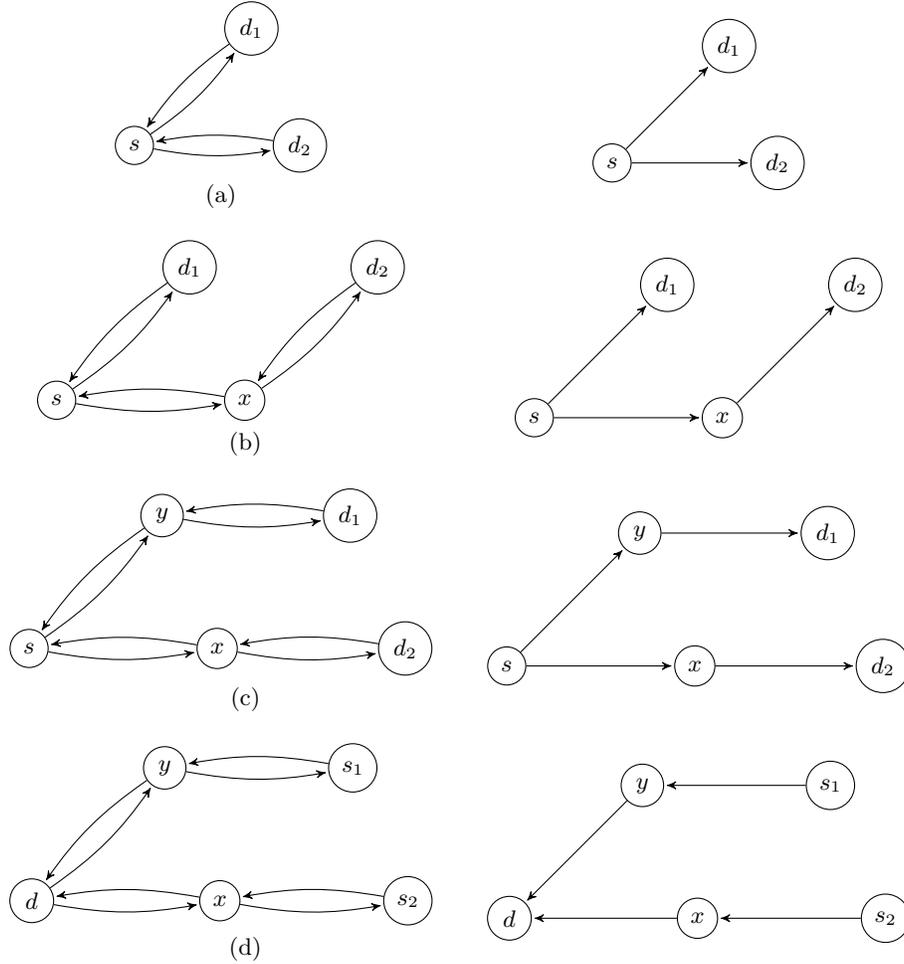


FIGURE 4. Existence of earliest arrival contraflow on multiple sinks

Theorem 4.6. *The EACF problem on $\mathcal{N} = (V, A, b_A, \tau, s, D, \mu(s), \nu(d), T)$ with zero transit times can be solved optimally.*

4.3. Approximate earliest arrival contraflow. So far, to the best of the author's knowledge, the EACF problem on multi-terminal networks has not been studied yet. From the observation of Examples 2 and 3 with arbitrary transit time and zero transit time, respectively, we can conclude that the EACF solution does not exist on multi-terminal network. Even in the case without contraflow, this problem has not been solved efficiently. However, authors in [11, 14] introduced an approximate EAF problem and presented time and value-approximate algorithms to solve the problem.

In this section, we introduce an approximate EACF problem on multi-terminal networks for the first time. We present a β -value-approximate algorithm to solve the problem that approximates the maximum amount of flow at each point in time with arc reversal capability. Moreover, we also present an efficient algorithm to solve the problem in case where each arc of network has zero transit time.

Problem 2. Given $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d), T)$, the problem is to compute a β -value-approximate EACF i.e., a dynamic flow x_{dyna} that achieves at every point in time $t \in \{1, \dots, T\}$ which is at least a β -fraction of the maximum dynamic flow value at time t if the direction of arcs can be reversed at time zero.

To solve Problem 2, we present Algorithm 2. Our algorithm is based on MDCF algorithm of [2, 24] for the MDCF problem and 2-value-approximate algorithm of [11, 14] for the approximate EAF problem.

Algorithm 2. 2-value-approximate EACF algorithm

1. Given a multi-terminal network $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \nu(d), T)$.
2. Construct the auxiliary network $\overline{\mathcal{N}} = (V, E, b_E, \tau, S, D, \mu(s), \nu(d), T)$ of \mathcal{N} .
3. Solve the EAF problem on network $\overline{\mathcal{N}}$ using algorithm of [11, 14].
4. Arc $(w, v) \in A$ is reversed, if and only if the flow along arc (v, w) is greater than $b_A(v, w)$ or if there is a nonnegative flow along arc $(v, w) \notin A$.
5. Obtain 2-value approximate EACF for the network \mathcal{N} .

We construct the auxiliary network $\overline{\mathcal{N}}$ in Step 2 of Algorithm 2 using the contraflow configuration. Then, we use the 2-value-approximate algorithm of [11, 14] in Step 3 that computes the 2-value-approximate EAF solution on multi-terminal auxiliary network $\overline{\mathcal{N}}$. It uses the standard time expanded network of [9] so that its time complexity is pseudo-polynomial. A maximum flow is computed by induction hypothesis in time expanded network. The computed solution with this algorithm is bounded by a factor of 2, i.e., $\beta = 2$ is the best possible factor for the algorithm as in [11, 14].

Theorem 4.7. For a multi-terminal network \mathcal{N} , a 2-value-approximate EACF solution always exists.

Proof. First we convert the network \mathcal{N} into an auxiliary network $\overline{\mathcal{N}}$. We know that the flow on the auxiliary network $\overline{\mathcal{N}}$ is feasible. The 2-value-approximation algorithm of [11, 14] calculates a 2-value-approximate EAF solution on $\overline{\mathcal{N}}$ that is equivalent to the 2-value-approximate EACF solution on \mathcal{N} . This solution is obtained on pseudo-polynomial time complexity. \square

For the special case of zero transit time, a 2-value-approximate EAF solution has been computed by replacing the Step 3 of Algorithm 2 with zero time 2-value-approximation algorithm of [11, 14] where the maximum static flow is computed and repeated them until a terminal runs out of demand/supply in polynomial time complexity. Thus we conclude with the following lemma.

Lemma 4.8. A 2-value-approximate EACF solution on \mathcal{N} with zero transit time can be computed in polynomial time complexity.

5. Conclusions. We solved the EACF problem efficiently on multiple sources and single sink networks with arbitrary transit time. For the special case of zero transit time, we obtained an EACF solution on single source and multiple sinks networks. We also presented an approximate EACF solution on multi-terminal networks for both cases of arbitrary transit time and zero transit time.

We solved these problems with the same complexity as without contraflow. However, we have realized that the flow values computed by contraflow models increase significantly that may be doubled for given time horizon. From the analytical point of view, we conjecture that if the dynamic cut capacities are symmetric, then these

networks compute double flow value for given time horizon with contraflow configuration. Moreover, for these networks, the time needed to transship given amount of flow value will be at most half with contraflow configuration.

To the best of our knowledge, these problems we introduced are for the first time in evacuation planning using discrete time setting. Moreover, we are very much interested to extend this model in continuous time setting.

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